

## L5 – 1.3 – Symmetry in Polynomial Functions

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In this section, you will learn about the properties of even and odd polynomial functions.

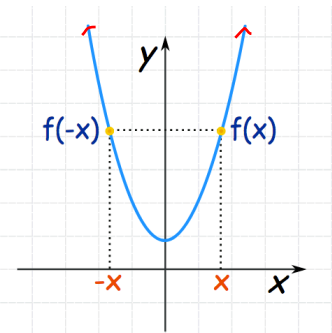
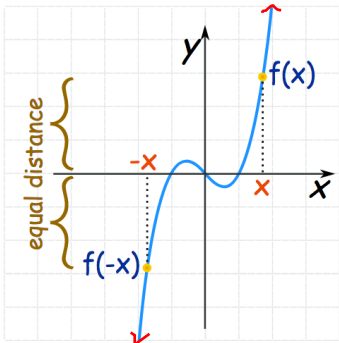
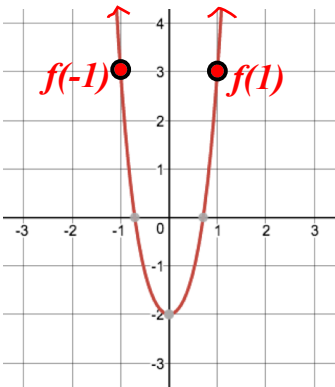
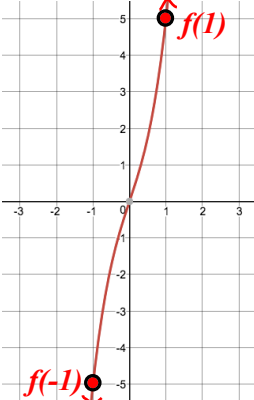
### Symmetry in Polynomial Functions

\_\_\_\_\_ – there is a vertical line over which the polynomial remains unchanged when reflected.

\_\_\_\_\_ – there is a point about which the polynomial remains unchanged when rotated  $180^\circ$

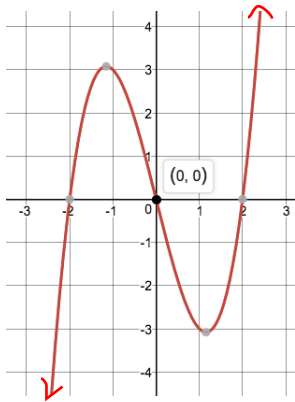
### Section 1: Properties of Even and Odd Functions

A polynomial function of even or odd degree is NOT necessarily an even or odd function. The following are properties of all even and odd functions:

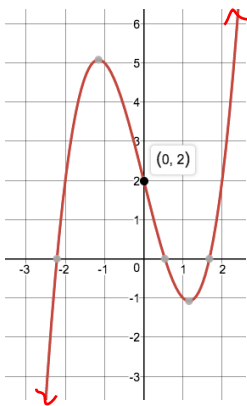
Even Functions	Odd Functions
<p>An even degree polynomial function is an <b>EVEN FUNCTION</b> if:</p> <ul style="list-style-type: none"> <li>• Line symmetry over the _____</li> <li>• The exponent of each term is _____</li> <li>• May have a constant term</li> </ul>	<p>An odd degree polynomial function is an <b>ODD FUNCTION</b> if:</p> <ul style="list-style-type: none"> <li>• Point symmetry about the _____</li> <li>• The exponent of each term is _____</li> <li>• No constant term</li> </ul>
<p>Rule:</p> 	<p>Rule:</p> 
<p>Example:</p>  <p><math>f(x) = 2x^4 + 3x^2 - 2</math></p> <p>Notice:</p> <p><math>f(1) =</math>  <math>f(-1) =</math></p> <p><math>\therefore</math></p>	<p>Example:</p>  <p><math>f(x) = 2x^3 + 3x</math></p> <p>Notice:</p> <p><math>f(1) =</math>  <math>f(-1) =</math></p> <p><math>\therefore</math></p>

**Example 1:** Identify each function as an even function, odd function, or neither. Explain how you can tell.

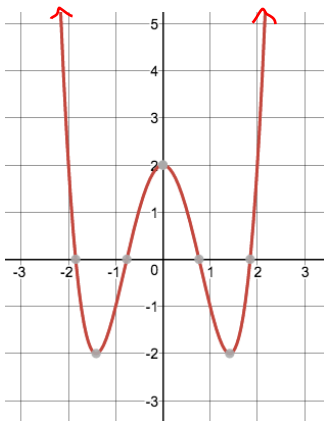
a)  $y = x^3 - 4x$



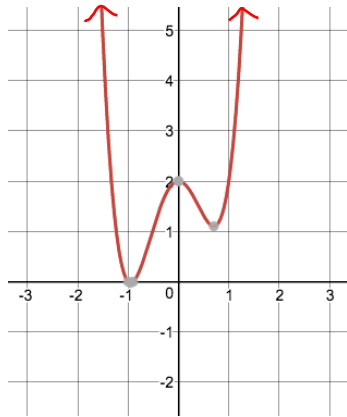
b)  $y = x^3 - 4x + 2$



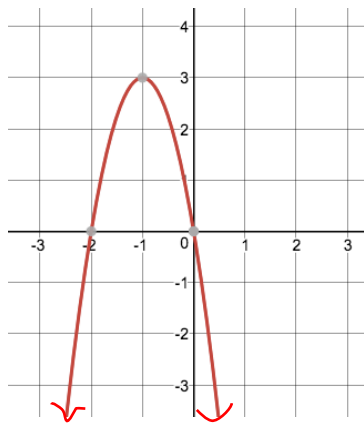
c)  $y = x^4 - 4x^2 + 2$



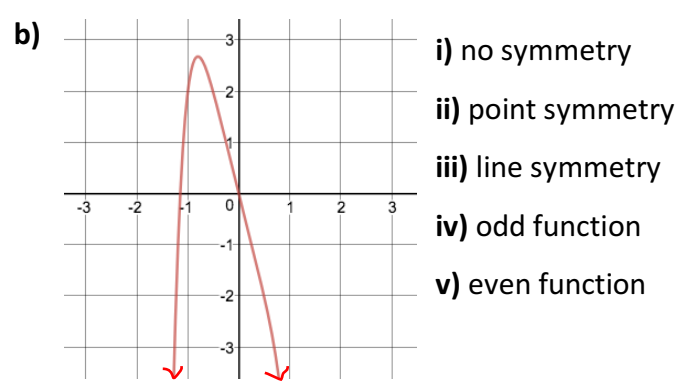
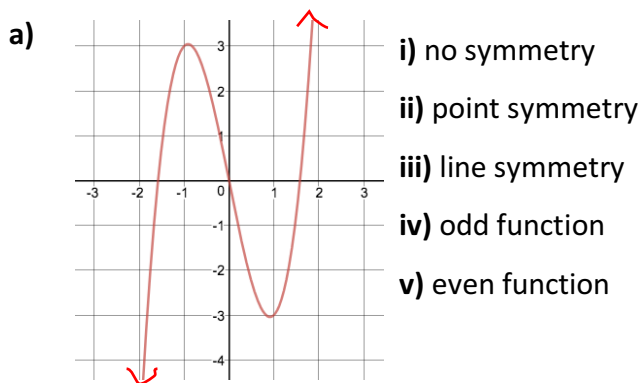
d)  $y = 3x^4 + x^3 - 4x^2 + 2$



e)  $y = -3x^2 - 6x$



**Example 2:** Choose all that apply for each function



c)  $P(x) = 5x^3 + 3x^2 + 2$

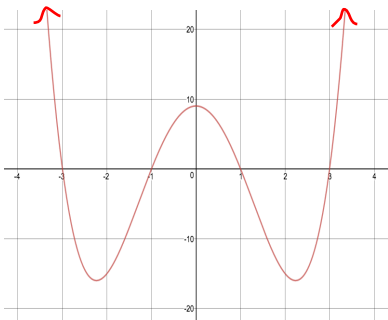
- i) no symmetry
- ii) point symmetry
- iii) line symmetry
- iv) odd function
- v) even function

**Note:**

d)  $P(x) = x^6 + x^2 - 11$

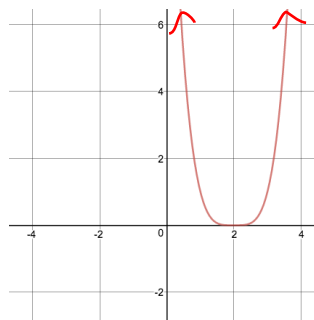
- i) no symmetry
- ii) point symmetry
- iii) line symmetry
- iv) odd function
- v) even function

e)



- i) no symmetry
- ii) point symmetry
- iii) line symmetry
- iv) odd function
- v) even function

f)



- i) no symmetry
- ii) point symmetry
- iii) line symmetry
- iv) odd function
- v) even function

g)  $P(x) = 5x^5 - 4x^3 + 8x$

- i) no symmetry
- ii) point symmetry
- iii) line symmetry
- iv) odd function
- v) even function

**Example 3:** Without graphing, determine if each polynomial function has line symmetry about the y-axis, point symmetry about the origin, or neither. Verify your response algebraically.

a)  $f(x) = 2x^4 - 5x^2 + 4$

b)  $f(x) = -3x^5 + 9x^3 + 2x$

c)  $x^6 - 4x^3 + 6x^2 - 4$

**Section 2: Connecting from throughout the unit**

**Example 4:** Use the given graph to state:

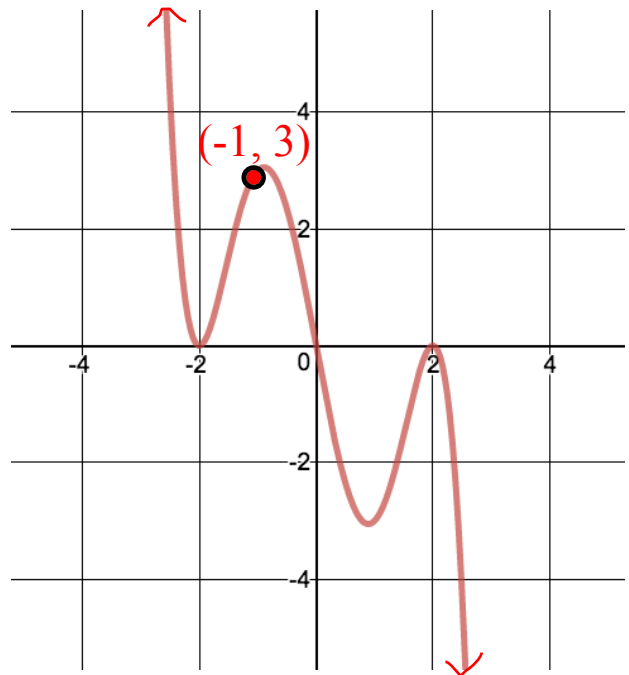
a)  $x$ -intercepts

b) number of turning points

c) least possible degree

b) any symmetry present

c) the intervals where  $f(x) < 0$



**d)** Find the equation in factored form