

## 2.1 Midpoint

### DO IT NOW

What is the slope of the following lines:

- The line connecting A(4,6) and B(12,10)
- Through the point A(4,3) and perpendicular to  $y = -2x - 7$
- Through point B(2,-6) and parallel to  $y = 3x - 8$

ANSWERS

### Unit 2: Chapter 2

### Analytic Geometry

### 2.1 Midpoint of a Line Segment

- 1) Complete Investigation Handout
- 2) Derive the midpoint formula
- 3) Examples
- 4) Classwork/Homework

Midpoint: point that divides a line segment into two equal line segments



## 2.1 Midpoint

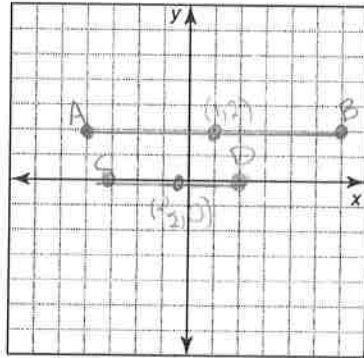
- a) A(-4,2) and B(6,2)    b) C(-3,0) and D(2,0)

Midpoint of AB

$(1, 2)$

Midpoint of CD

$(-\frac{1}{2}, 0)$



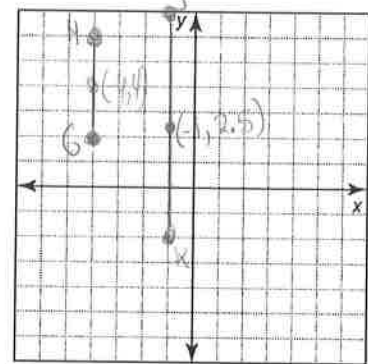
- b) G(-4,2) and H(-4,6)    b) J(-1,7) and K(-1,-2)

Midpoint of GH

$(-4, 4)$

Midpoint of JK

$(-1, 2.5)$



- c) R(-5,-4) and S(-1,0)    b) V(-4,6) and W(3,4)

Midpoint of RS

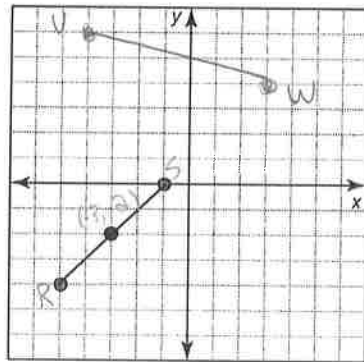
$(-3, -2)$

Midpoint of VW

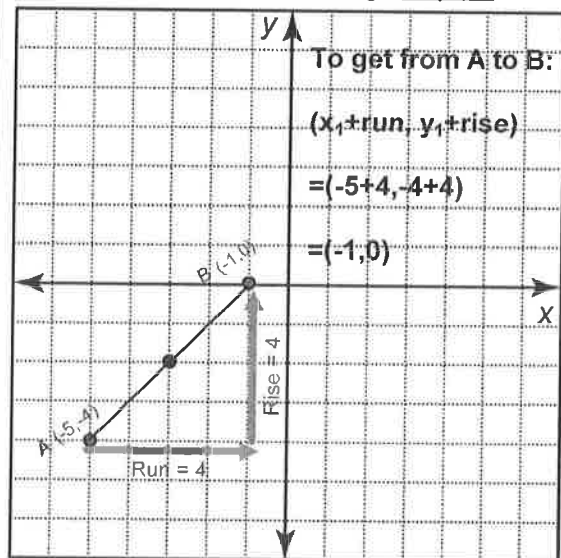
$(\quad, \quad)$



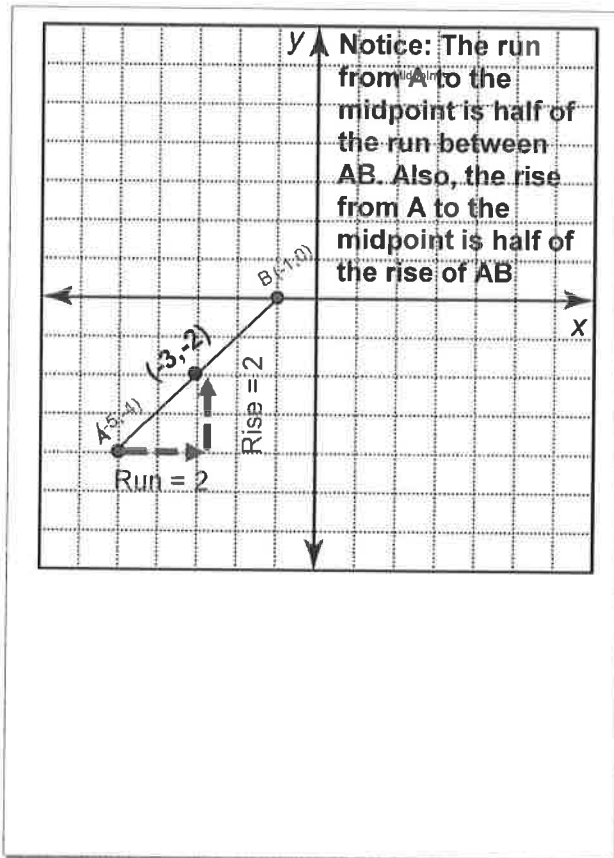
Need The FORMULA!



### Rise and Run Method of Finding a Midpoint:



## 2.1 Midpoint



There is an easier way to find the midpoint!!!!

### Note:

The middle of 2 points

=

The average of the 2 points

**Don't Forget:** Each point has an x and y coordinate. Therefore we must find the average (middle) of each coordinate to find the midpoint of the line joining the points.

Endpoints:  $A(x_1, y_1) = (-5, -4)$   $B(x_2, y_2) = (-1, 0)$

Midpoint:  $(-3, -2)$

Notice:

- the x-coordinate of the midpoint is the mean (average) of the x-coordinates of the endpoints of AB
- the y-coordinate of the midpoint is the mean (average) of the y-coordinates of the endpoints of AB

average of x-coordinates:

$$\begin{aligned} & \frac{x_1 + x_2}{2} \\ &= \frac{(-5) + (-1)}{2} \\ &= \frac{-6}{2} \\ &= -3 \end{aligned}$$

average of y-coordinates:

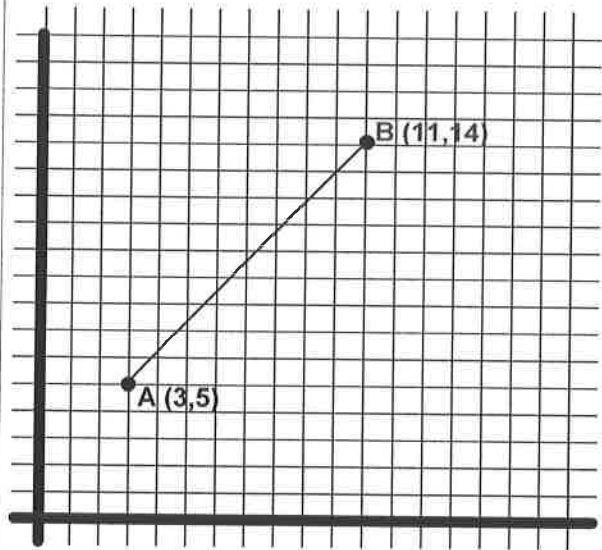
$$\begin{aligned} & \frac{y_1 + y_2}{2} \\ &= \frac{(-4) + (0)}{2} \\ &= \frac{-4}{2} \\ &= -2 \end{aligned}$$

## MIDPOINT FORMULA

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

## 2.1 Midpoint

**Example 1:** Calculate Midpoint Using the Formula



$$\begin{array}{cc} x_1 & y_1 & x_2 & y_2 \\ A(3,5) & & B(11,14) & \end{array}$$

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{3+11}{2}, \frac{5+14}{2} \right)$$

$$= \left( \frac{14}{2}, \frac{19}{2} \right)$$

$$= \left( 7, \frac{19}{2} \right)$$

**Example 2:** Calculate Midpoint Using the Formula

$$\begin{array}{cc} x_1 & y_1 & x_2 & y_2 \\ A(5,7) & & B(3,9) & \end{array}$$

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{5+3}{2}, \frac{7+9}{2} \right)$$

$$= \left( \frac{8}{2}, \frac{16}{2} \right)$$

$$= (4, 8)$$

**Example 3:** Determine the coordinates of the midpoint of the line segment with endpoints:

A(-1,0) and B(1,-6)

$$\begin{array}{cc} x_1 & y_1 & x_2 & y_2 \end{array}$$

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{-1+1}{2}, \frac{0+(-6)}{2} \right)$$

$$= \left( \frac{0}{2}, \frac{-6}{2} \right)$$

$$= (0, -3)$$

## 2.1 Midpoint

**Example 4:** Determine the coordinates of the midpoint of the line segment with endpoints:

A (3,-3) and B (7,-5)

$x_1$   $y_1$        $x_2$   $y_2$

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{3+7}{2}, \frac{-3+(-5)}{2} \right)$$

$$= \left( \frac{10}{2}, \frac{-8}{2} \right)$$

$$= (5, -4)$$

**Example 5:** Determine the coordinates of the midpoint of the line segment with endpoints:

A (6,-1) and B (-3,7)

$x_1$   $y_1$        $x_2$   $y_2$

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{6+(-3)}{2}, \frac{-1+7}{2} \right)$$

$$= \left( \frac{3}{2}, \frac{6}{2} \right)$$

$$= \left( \frac{3}{2}, 3 \right)$$

**Example 6:** Determine the coordinates of the midpoint of the line segment with endpoints:

A (-5/8, 1/8) and B (4, 6/8)

$x_1$   $y_1$        $x_2$   $y_2$

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

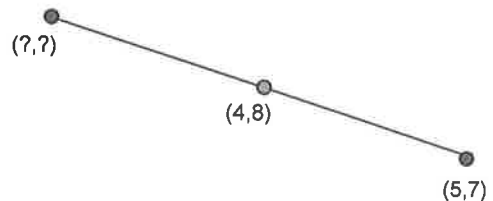
$$= \left( \frac{-\frac{5}{8} + 4}{2}, \frac{\frac{1}{8} + \frac{6}{8}}{2} \right)$$

$$= \left( \frac{-\frac{5}{8} + \frac{32}{8}}{2}, \frac{\frac{7}{8}}{2} \right)$$

$$= \left( \frac{\frac{27}{8}}{2}, \frac{\frac{7}{8}}{2} \right)$$

$$= \left( \frac{27}{16}, \frac{7}{16} \right)$$

What if you are given the midpoint, one endpoint, and are missing the other endpoint?



## 2.1 Midpoint

**Example 7:** Find the other endpoint of the line segment with the given endpoint and midpoint

**Endpoint: (5,7) Midpoint: (4,8)**

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(4,8) = \left( \frac{5 + x_2}{2}, \frac{7 + y_2}{2} \right)$$

$$(4,8) = \left( \frac{5 + x_2}{2}, \frac{7 + y_2}{2} \right)$$

$$4 = \frac{5 + x_2}{2}$$

$$8 = \frac{7 + y_2}{2}$$

$$8 = 5 + x_2$$

$$16 = 7 + y_2$$

$$3 = x_2$$

$$9 = y_2$$

The other endpoint is:

$$(3, 9)$$

**Example 8:** Find the other endpoint of the line segment with the given endpoint and midpoint

**Endpoint: (-1,9) Midpoint: (-9,-10)**

$$\text{MIDPOINT} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(-9, -10) = \left( \frac{-1 + x_2}{2}, \frac{9 + y_2}{2} \right)$$

$$-9 = \frac{-1 + x_2}{2}$$

$$-10 = \frac{9 + y_2}{2}$$

$$-18 = -1 + x_2$$

$$-20 = 9 + y_2$$

$$-17 = x_2$$

$$-29 = y_2$$

$$\text{endpoint } (-17, -29)$$

Homework:

Complete Worksheet  
and  
Pg. 66 #2,3