L3 – 2.2 – Factor Theorem Lesson
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In this section, you will learn how to determine the factors of a polynomial function of degree 3 or greater.

Part 1: Remainder Theorem Refresher

a) Use the remainder theorem to determine the remainder when $f(x) = x^3 + 4x^2 + x - 6$ is divided by $x + 2$

Remainder Theorem: When a polynomial function $P(x)$ is divided by $x - b$, the remainder is $P(b)$; and when it is divided by $ax - b$, the remainder is $P\left(\frac{b}{a}\right)$, where a and b are integers, and $a \neq 0$.

b) Verify your answer to part a) by completing the division using long division or synthetic division.

Note: I chose synthetic since it is a linear divisor of the form $x - b$.

Factor Theorem:

$x - b$ is a factor of a polynomial $P(x)$ if and only if $P(b) = 0$. Similarly, $ax - b$ is a factor of $P(x)$ if and only if $P\left(\frac{b}{a}\right) = 0$.

Example 1: Determine if $x - 3$ and $x + 2$ are factors of $P(x) = x^3 - x^2 - 14x + 24$

$P(3) =$

Since the remainder is ___, $x - 3$ divides evenly into $P(x)$; that means $x - 3$ ____________________ of $P(x)$.

$P(-2) =$

Since the remainder is not ____, $x + 2$ does not divide evenly into $P(x)$; that means $x + 2$ ________________________ of $P(x)$.

Part 2: How to determine a factor of a Polynomial With Leading Coefficient 1

You could guess and check values of b that make $P(b) = 0$ until you find one that works…

Or you can use the Integral Zero Theorem to help.

Integral Zero Theorem

If $x - b$ is a factor of a polynomial function $P(x)$ with leading coefficient 1 and remaining coefficients that are integers, then ***b* is a factor of the constant term** of $P(x)$.

Note: Once one of the factors of a polynomial is found, division is used to determine the other factors.

Example 2: Factor $x^3 + 2x^2 - 5x - 6$ fully.

Let $P(x) = x^3 + 2x^2 - 5x - 6$

Find a value of b such that $P(b) = 0$. Based on the factor theorem, if $P(b) = 0$, then we know that $x - b$ is a factor. We can then divide $P(x)$ by that factor.

The integral zero theorem tells us to test factors of _____

Test ________________________________. Once one factor is found, you can stop testing and use that factor to divide $P(x)$.

$P(1) =$

Since ________________, we know that ________________________ a factor of $P(x)$.

$P(2) =$

Since ________________, we know that ____________________ a factor of $P(x)$.

You can now use either long division or synthetic division to find the other factors

Method 1: Long division

Method 2: Synthetic Division

Example 3: Factor $x^4 + 3x^3 - 7x^2 - 27x - 18$ completely.

Let $P(x) = x^4 + 3x^3 - 7x^2 - 27x - 18$

Find a value of b such that $P(b) = 0$. Based on the factor theorem, if $P(b) = 0$, then we know that $x - b$ is a factor. We can then divide $P(x)$ by that factor.

The integral zero theorem tells us to test factors of ________

Test ____. Once one factor is found, you can stop testing and use that factor to divide $P(x)$.

Since ________________, this tell us that ____________ is a factor. Use division to determine the other factor.

We can now further divide $x^3 + 2x^2 - 9x - 18$ using division again or by factoring by grouping.

Method 1: Division

Method 2: Factoring by Grouping

Group the first 2 terms and the last 2 terms and separate with an addition sign.

Common factor within each group

Factor out the common binomial

Therefore,

$x^4 + 3x^3 - 7x^2 - 27x - 18 =$

Example 4: Try Factoring by Grouping Again

$x^4 - 6x^3 + 2x^2 - 12x$

Note: Factoring by grouping does not always work…but when it does, it saves you time!

Part 3: How to determine a factor of a Polynomial With Leading Coefficient NOT 1

The integral zero theorem can be extended to include polynomials with leading coefficients that are not 1. This extension is known as the rational zero theorem.

Rational Zero Theorem:

Suppose $P(x)$ is a polynomial function with integer coefficients and $x = \frac{b}{a}$ is a zero of $P(x)$, where a and b are integers and $a \neq 0$. Then,

- b is a factor of the constant term of $P(x)$
- a is a factor of the leading coefficient of $P(x)$
- $(ax - b)$ is a factor of $P(x)$

Example 5: Factor $P(x) = 3x^3 + 2x^2 - 7x + 2$

We must start by finding a value of $\frac{b}{a}$ where $P\left(\frac{b}{a}\right) = 0$.

b must be a factor of the constant term. Possible values for b are: ___________

a must be a factor of the leading coefficient. Possible values of a are: ___________

Therefore, possible values for $\frac{b}{a}$ are: _____________________________

Test values of $\frac{b}{a}$ for x in $P(x)$ to find a zero.

Since ____ of $P(x)$. Use division to find the other factors.

Example 6: Factor $P(x) = 2x^3 + x^2 - 7x - 6$

Possible values for b are: ________________________

Possible values of a are: ____________

Therefore, possible values for $\frac{b}{a}$ are: ____________________________________

Part 4: Application Question

Example 7: When $f(x) = 2x^3 - mx^2 + nx - 2$ is divided by $x + 1$, the remainder is -12 and $x - 2$ is a factor. Determine the values of m and n.

Hint: Use the information given to create 2 equations and then use substitution or elimination to solve.