

W4 - 2.3 - Solving Polynomial Equations

MHF4U

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1) Determine the solutions of the following polynomials.

a) $(3x + 2)(x + 9)(x - 2) = 0$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 3x+2=0 & x+9=0 & x-2=0 \\ x_1 = -\frac{2}{3} & x_2 = -9 & x_3 = 2 \end{array}$$

$(-\frac{2}{3}, 0), (-9, 0), (2, 0)$

b) $(x^2 + 1)(x - 4) = 0$

$$\begin{array}{cc} \downarrow & \downarrow \\ x^2+1=0 & x-4=0 \\ x^2=-1 & x=4 \\ \text{No Solutions} & \end{array}$$

$(4, 0)$

2) Determine the solutions of the following polynomials by factoring. Use the tools you have learned this unit to help you. (remainder theorem, integral zero theorem, division etc.)

a) $x^3 - 4x^2 - 3x + 18 = 0$

Possible Factors: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$f(-2) = 0$; so $x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -3 & 18 \\ & \downarrow & & & \\ & & -2 & 12 & -18 & + \\ \hline x & 1 & -6 & 9 & 0 \\ & x^2 & x & + & R \end{array}$$

$(x+2)(x^2 - 6x + 9) = 0$

$(x+2)(x-3)^2 = 0$

$$\begin{array}{cc} \downarrow & \downarrow \\ x+2=0 & x-3=0 \end{array}$$

$x_1 = -2$ $x_2 = 3$

Solutions: $(-2, 0)$ and $(3, 0)$

b) $x^3 - 3x^2 - 4x + 12 = 0$

Possible Factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$f(2) = 0$; so $x-2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & \downarrow & & & \\ & & 2 & -2 & -12 & + \\ \hline x & 1 & -1 & -6 & 0 \\ & x^2 & x & + & R \end{array}$$

$(x-2)(x^2 - x - 6) = 0$

$(x-2)(x-3)(x+2) = 0$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x-2=0 & x-3=0 & x+2=0 \\ \boxed{x_1=2} & \boxed{x_2=3} & \boxed{x_3=-2} \end{array}$$

Solutions: $(2, 0), (3, 0),$ and $(-2, 0)$

$$c) x^4 - x^3 - 11x^2 + 9x + 18 = 0$$

Possible Factors: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$f(-1) = 0$; $\therefore x+1$ is a factor

$$\begin{array}{r|rrrrrr} -1 & 1 & -1 & -11 & 9 & 18 \\ & & \downarrow & & & & + \\ x & 1 & -2 & -9 & 18 & 0 \\ & x^3 & x^2 & x & \# & R \end{array}$$

$$(x+1)(x^3 - 2x^2 - 9x + 18) = 0$$

$$(x+1)[x^2(x-2) - 9(x-2)] = 0$$

$$(x+1)(x-2)(x^2-9) = 0$$

$$(x+1)(x-2)(x-3)(x+3) = 0$$

$$x_1 = -1 \quad x_2 = 2 \quad x_3 = 3 \quad x_4 = -3$$

Solutions:

$$(-1, 0), (2, 0), (3, 0), \text{ and } (-3, 0)$$

$$e) 2x^3 - 7x^2 + 10x - 5 = 0$$

Possible Factors: $\pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}$

$f(1) = 0$; $\therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & -7 & 10 & -5 \\ & & \downarrow & & & + \\ x & 2 & -5 & 5 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$(x-1)(2x^2 - 5x + 5) = 0$$

\downarrow

$$x-1=0$$

$$x=1$$

$$\downarrow \text{ check } b^2 - 4ac = (-5)^2 - 4(2)(5) = -15$$

\therefore No solutions

$$\text{Solution: } (1, 0)$$

$$d) x^3 - 64 = 0$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(x-4)(x^2 + 4x + 16) = 0$$

\downarrow

$$x=4$$

$$\downarrow \text{ check } b^2 - 4ac = (4)^2 - 4(1)(16) = -48$$

No solutions

$$\text{Solution: } (4, 0)$$

3) Solve each equation by first factoring the sum or difference of cubes.

a) $x^3 - 8 = 0$

$$(x-2)(x^2+2x+4) = 0$$

$$\downarrow$$

$$x-2=0$$

$$\boxed{x=2}$$

$$\downarrow$$

check $b^2-4ac = (2)^2 - 4(1)(4)$
 $= -12$
 \therefore No solution

$$\boxed{\text{Solution: } (2, 0)}$$

b) $x^3 + 27 = 0$

$$(x+3)(x^2-3x+9) = 0$$

$$\downarrow$$

$$x+3=0$$

$$\boxed{x=-3}$$

$$\downarrow$$

check $b^2-4ac = (-3)^2 - 4(1)(9)$
 $= -27$
 \therefore No solution

$$\boxed{\text{Solution: } (-3, 0)}$$

4) Solve by factoring

a) $x^3 - 4x^2 - 7x + 10 = 0$

Possible Factors: $\pm 1, \pm 2, \pm 5, \pm 10$

$f(1) = 0$; $\therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & \downarrow & & & \\ x & 1 & -3 & -10 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$(x-1)(x^2-3x-10) = 0$$

$$(x-1)(x-5)(x+2) = 0$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$x-1=0 \quad x-5=0 \quad x+2=0$$

$$\boxed{x_1=1} \quad \boxed{x_2=5} \quad \boxed{x_3=-2}$$

$$\boxed{\text{Solutions: } (1, 0), (5, 0), \text{ and } (-2, 0)}$$

b) $2x^3 - 11x^2 + 12x + 9 = 0$

Possible Factors: $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

$f(3) = 0$; $\therefore x-3$ is a factor

$$\begin{array}{r|rrrr} 3 & 2 & -11 & 12 & 9 \\ & \downarrow & & & \\ x & 2 & -5 & -3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$(x-3)(2x^2-5x-3) = 0$$

$$(x-3)(x-3)(2x+1) = 0$$

$$(x-3)^2(2x+1) = 0$$

$$\downarrow$$

$$x-3=0 \quad 2x+1=0$$

$$\boxed{x_1=3} \quad \boxed{x_2=-\frac{1}{2}}$$

$$\boxed{\text{Solutions: } (3, 0) \text{ and } (-\frac{1}{2}, 0)}$$



$$c) x^4 - x^3 - 2x - 4 = 0$$

Possible factors: $\pm 1, \pm 2, \pm 4$

$f(-1) = 0$; $x+1$ is a factor

$$\begin{array}{r|rrrrr}
 -1 & 1 & -1 & 0 & -2 & -4 \\
 & \downarrow & & & & \\
 x & 1 & -2 & 2 & -4 & 0 \\
 & x^3 & x^2 & x & \# & R
 \end{array}$$

$$(x+1)(x^3 - 2x^2 + 2x - 4) = 0$$

$$(x+1)[x^2(x-2) + 2(x-2)] = 0$$

$$(x+1)(x-2)(x^2+2) = 0$$

$$\begin{array}{ccc}
 \downarrow & \downarrow & \downarrow \\
 x+1=0 & x-2=0 & x^2+2=0 \\
 \boxed{x_1 = -1} & \boxed{x_2 = 2} & \text{No solution}
 \end{array}$$

$$\boxed{\text{Solutions: } (-1, 0) \text{ and } (2, 0)}$$

ANSWER KEY

1a) $(-\frac{2}{3}, 0), (-9, 0), (2, 0)$ b) $(4, 0)$

2a) $(-2, 0)$ and $(3, 0)$ b) $(3, 0), (-2, 0), (2, 0)$ c) $(-1, 0), (2, 0), (-3, 0), (3, 0)$ d) $(4, 0)$ e) $(1, 0)$

3a) $(2, 0)$ b) $(-3, 0)$

4a) $(5, 0), (-2, 0), (1, 0)$ b) $(-0.5, 0)$ and $(3, 0)$ c) $(-1, 0)$ and $(2, 0)$