2.6 - Inverse of a Function

Inverse of a function:

- The inverse of a function f is denoted as f^{-1}
- The function and its inverse have the property that if $f(a) = b$, then $f^{-1}(b) = a$
- So if $f(5) = 13$, then $f^{-1}(13) = 5$.

- More simply put: The inverse of a function has all the same points as the original function, except that the x's and y's have been reversed.

It is important to note that $f^{-1}(x)$ is read as "the inverse of f at x". The -1 does not behave like an exponent.

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

To draw an inverse, all you need to do is swap the x and y coordinates of each point.

Finding Inverses by Numerically

Example 1: The table shows ordered pairs belonging to a function $f(x)$. Determine $f^{-1}(x)$, then state the domain and range of $f(x)$ and its inverse.

$f(x)$	$f^{-1}(x)$
(-5, 0)	(0, -5)
(-4, 2)	(2, -4)
(-3, 5)	(5, -3)
(-2, 6)	(6, -2)
(0, 7)	(7, 0)

$f(x)$

D: $\{x \in \mathbb{R} \mid x = -5, -4, -3, -2, 0\}$

R: $\{y \in \mathbb{R} \mid y = 0, 2, 5, 6, 7\}$

$f^{-1}(x)$

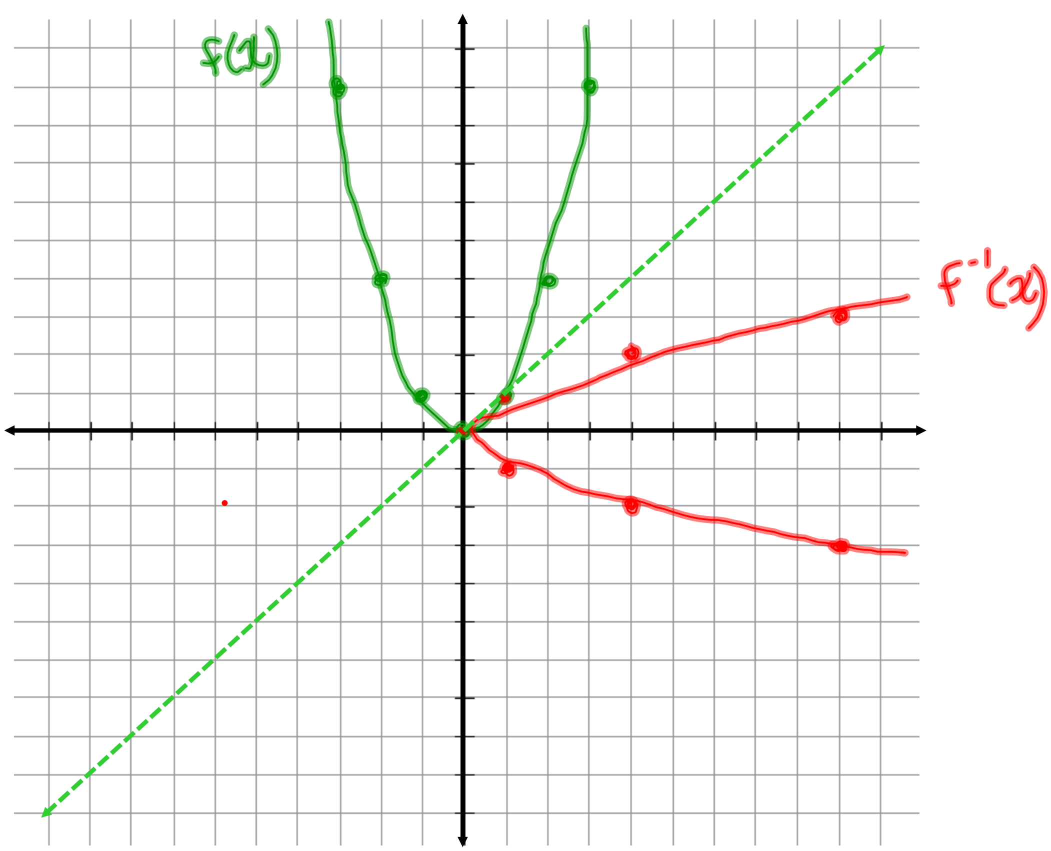
D: $\{x \in \mathbb{R} \mid x = 0, 2, 5, 6, 7\}$

R: $\{y \in \mathbb{R} \mid y = -5, -4, -3, -2, 0\}$

Example 2:

a) Graph the function $f(x) = x^2$ and its inverse $f^{-1}(x)$

$f(x)$	$f^{-1}(x)$
(-3,9)	(9,-3)
(-2,4)	(4,-2)
(-1,1)	(1,-1)
(0,0)	(0,0)
(1,1)	(1,1)
(2,4)	(4,2)
(3,9)	(9,3)



b) state the domain and range of both functions

$f(x)$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

$f^{-1}(x)$

D: $\{x \in \mathbb{R} \mid x \geq 0\}$

R: $\{y \in \mathbb{R}\}$

***Note:** the domain and range of inverse functions are the reverse of each other.*

Example 3:

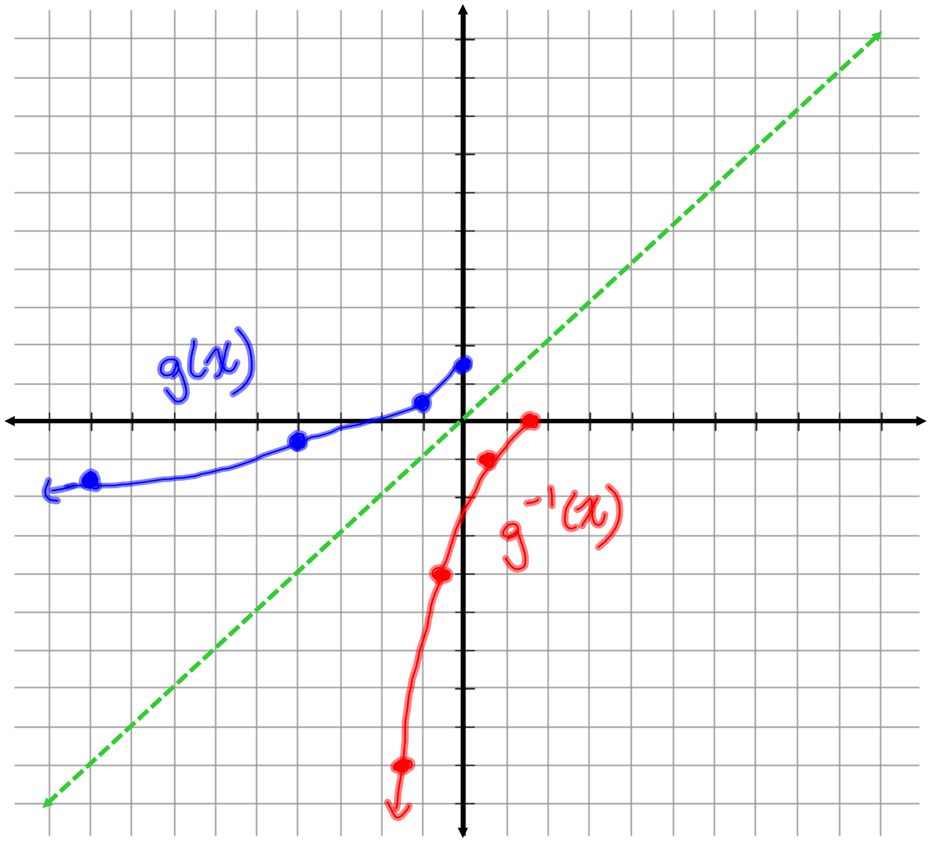
Sketch the graph of $g(x) = -2\sqrt{(-\frac{1}{2} x)} + 3$ then graph $g^{-1}(x)$.

$f(x) = \sqrt{x}$ ⟶ $g(x) = -2\sqrt{(-\frac{1}{2}x)} + 3$ ⟶ $g^{-1}(x)$

x	*y*
0	0
1	1
4	2
9	3

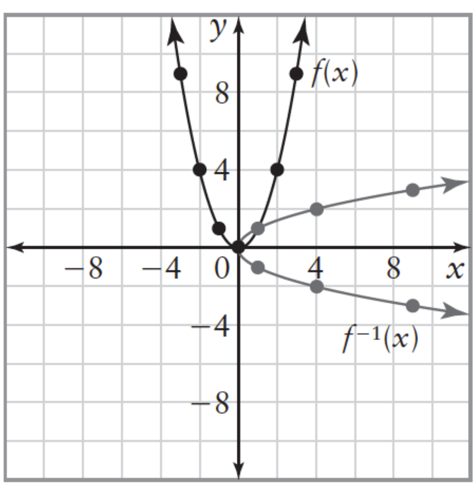
$-2x$	$-2y+3$
0	3
-2	1
-8	-1
-18	-3

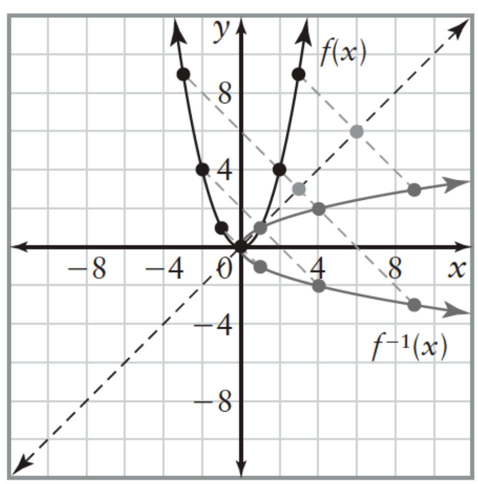
x	*y*
3	0
1	-2
-1	-8
-3	-18



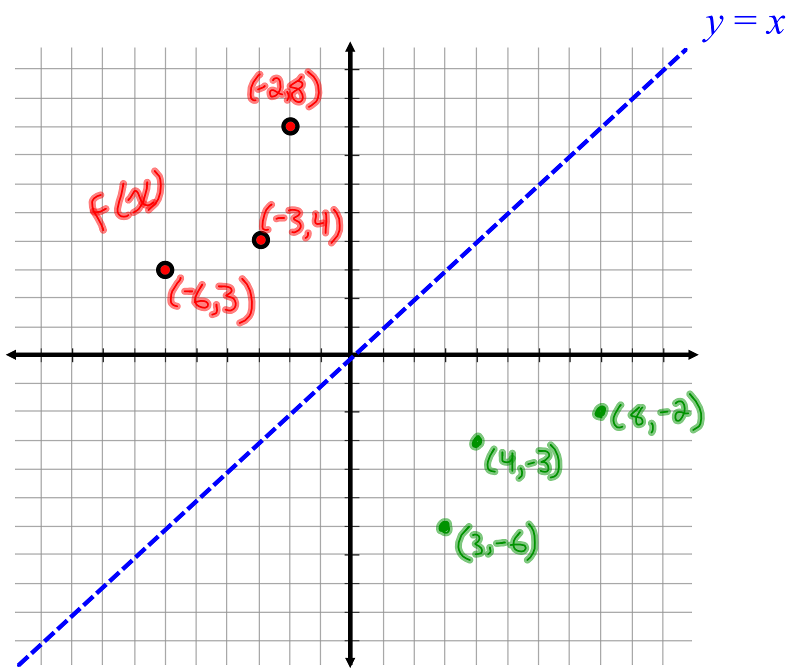
Finding Inverses by Graphing

The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$. This is true for all functions and their inverses. If you find the midpoint of each pair of points from example 2 and connect them you can prove this theorem.





Example 4: Sketch the inverse of the $f(x)$



Finding Inverses Algebraically

Algebraic Method for finding the inverse:

1. Replace $f(x)$ with "y"
2. Switch the x and y variables
3. Isolate for y
4. replace y with $f^{-1}(x)$

Example 5: Find the inverse of the following functions…

a) $g(x) = \frac{(3x)}{4}$

$y = \frac{3x}{4}$

$x = \frac{3y}{4}$

$4x = 3y$

$\frac{4x}{3} = y$

$g^{-1}(x) = \frac{4x}{3}$

b) $h(x) = 4x + 3$

$y = 4x + 3$

$x = 4y + 3$

$x - 3 = 4y$

$\frac{x-3}{4} = y$

$h^{-1}(x) = \frac{x-3}{4}$

c) $f(x) = x^2 - 1$

$y = x^2 - 1$

$x = y^2 - 1$

$x + 1 = y^2$

$\pm\sqrt{x+1} = y$

$f^{-1}(x) = \pm\sqrt{x+1}$

d) $h(x) = \frac{4x+3}{5}$

$y = \frac{4x+3}{5}$

$x = \frac{4y+3}{5}$

$5x = 4y + 3$

$\frac{5x-3}{4} = y$

$h^{-1}(x) = \frac{5x-3}{4}$

e) $f(x) = 2x^2 + 16x + 29$

Note: for algebraic inverses of quadratic functions, before interchanging x and y's you must re-write in vertex form.

$y = (2x^2 + 16x) + 29$

$y = 2(x^2 + 8x + 16 - 16) + 29$

$y = 2(x^2 + 8x + 16) - 32 + 29$

$y = 2(x+4)^2 - 3$

$x = 2(y+4)^2 - 3$

$\frac{x+3}{2} = (y+4)^2$

$\pm\sqrt{\frac{x+3}{2}} = y + 4$

$-4 \pm \sqrt{\frac{x+3}{2}} = y$

$f^{-1}(x) = -4 \pm \sqrt{\frac{x+3}{2}}$

f) $r(x) = \sqrt{(x)} + 2$

$y = \sqrt{x} + 2$

$x = \sqrt{y} + 2$

$x - 2 = \sqrt{y}$

$(x-2)^2 = y$

$r^{-1}(x) = (x-2)^2$