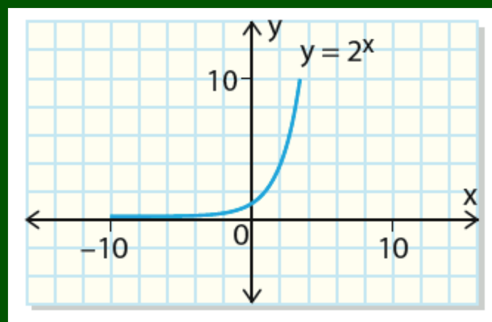
3.1 Exponential Growth



http://www.youtube.com/watch?v=oFyo9O5-YnM&feature=youtu.be

DO IT NOW!

A type of bacteria grows so that it triples in number every day.
On the day we begin observations, the bacteria has a population of 100.

a) Make a table to show the population over 5 days.

Day	Population
0	100
1	100×3 = 300
2	300×3 = 900
3	2700
4	8100
5	24300

b) Calculate finite differences and indicate any patterns you see

Day	Population	1st Differences	2nd Differences
0	100		
1	300	300-100 = 200	
2	900	900-300=600	400
3	2700	2700 - 900 = 1800	1200
4	8100	8100 - 2700 = 5400	3600
5	24 300	24300-8100= 16 200	10800
		×3	×3

If the finite differences of a set of data increase by a constant multiple, it is an **<u>exponential</u>** relation.

c) Graph the relation

d) Write an equation to model this growth

Day	Population
0	100
1	$100 \times 3^1 = 300$
2	$100 \times 3^2 = 900$
3	$100 \times 3^3 = 2700$
4	$100 \times 3^4 = 8100$

The relationship between days and total population is easier to see when we look at the number of times the population has been tripled.

$$y = 100(3^x)$$

y = total population

x = number of days

100 = initial population

3 because the population triples each day

General Properties of Exponential Growth

Equation: $y = a(b^x)$

a = initial amount or population

b = growth factor ($b > 1$)

y = future amount or population

x = number of times 'a' has increased

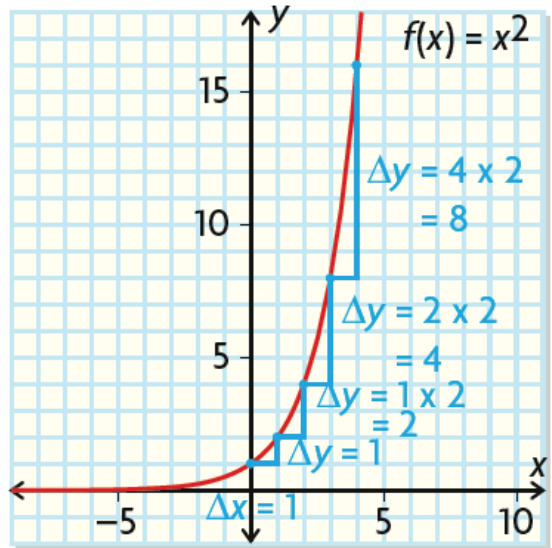
To calculate 'x' use equation →

$$x = \frac{\text{total time}}{\text{time it takes for one growth period}}$$

Graph of an exponential growth function:

Exponential growth is represented by a quickly increasing curve.

First (and second) differences are related by multiplication



Example 1:

Your brother tells you a secret. You see no harm in telling two friends. After this second "passing" of the secret, 4 people now know the secret (your brother, you and two friends). If each of these friends now tells two new people, after the third "passing" of the secret, eight people will know. If this pattern of spreading the secret continues, how many people will know the secret after 10 such "passings"?

$a = 1$
$b = 2$

$$y = 1(2^x)$$

$$y = 2^{10}$$

$$y = 1024$$

On the 10th passing, 1024 people will know the secret.

Example 2:

a) An insect colony has a current population of 50 insects. Its population doubles every 3 days. What is the population after 12 days?

$a = 50$

$b = 2$

$y = 50(2^x) \rightarrow x = \frac{12}{3} = 4$

$y = 50(2^4)$

$y = 50(16)$

$y = 800$

The population will be 800 after 12 days.

b) The insect colony is actually full of giant, intelligent, mutant insects. They plot that they can overtake the Earth when their population has reached 1 billion. When will we meet our doom? (When does the population reach 1 billion?)

$y = 50(2^x)$

$1\,000\,000\,000 = 50(2^x)$

$20\,000\,000 = 2^x$

$\log 20\,000\,000 = \log 2^x$

$\log 20\,000\,000 = x(\log 2)$

$\frac{\log 20\,000\,000}{\log 2} = x$

$x = 24.25$

but remember, $x = \frac{t}{3}$

$24.25 = \frac{t}{3}$

$t = 72.76$

We should be prepared for the insects to take over in 72.76 days.

If exponential growth is given as a percent use the equation:

$$y = a(1+r)^x$$

a = initial amount

r = growth rate (given as percent)

x = number of times 'a' has been increased

Example 3:

In 2005, there were only 285 Pittsburgh Penguins fans in Oakville. The number of Penguins fans increased by 75% per year after 2005 (this is when Crosby was drafted). How many Crosby fans are now in Oakville in 2014?

$a = 285$

$r = 0.75$

$y = 285(1.75)^x$

$y = 285(1.75)^9$

$y = 43871.99$

In 2014 their are approximately 43 872 Pittsburgh fans.