

3.1 Exponential Growth - Worksheet

MCR3U

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SOLUTIONS

1) An insect colony, with an initial population of 20, quadruples every day.

a) Complete the table.

Day	Population	First Differences	Second Differences
0	20		
1	80	60	180
2	320	240	720
3	1280	960	2880
4	5120	3840	11520
5	20480	15360	

b) Is the relationship between the insect population and the number of days exponential? Explain how you can tell.

The values in each difference column increase by a factor of 4; so it is exponential.

2) Suppose that there is a rumour going around your school that next year all weekends will be extended to three days. Initially, on day 0, five students know the rumour. Suppose that each person who knows the rumour tells two more students the day after they hear about it. Also assume that no-one hears the rumour more than once.

How many people will learn about the rumour...

i) on day 1

$$\text{Day 1} = 5 \times 2 = 10$$

ii) on day 2

$$\text{Day 2} = 10 \times 2 = 20$$

3) A bacteria colony has an initial population of 200. The population triples every week.

a) Write an equation to relate population, p , to time, t , in weeks.

$$p = 200(3)^t$$

b) Determine the approximate population after 10 days.

$$p = 200(3)^{10/7}$$

$$p = 960.8$$

Approximately 961

c) Determine the approximate population after 12 weeks.

$$\begin{aligned} p &= 200(3)^{12} \\ &= 106\ 288\ 200 \end{aligned}$$

4) An investment of \$4000 earns interest, which causes it to increase by 6% per year.

a) Write an equation for the amount, A , in dollars, that the investment is worth as a function of time, n , in years.

$$A = 4000(1.06)^n$$

b) Determine the amount that the investment is worth after

i) 2 years

$$\begin{aligned} A &= 4000(1.06)^2 \\ &= \$4494.40 \end{aligned}$$

ii) 10 years

$$\begin{aligned} A &= 4000(1.06)^{10} \\ &= \$7163.39 \end{aligned}$$

iii) 20 years

$$\begin{aligned} A &= 4000(1.06)^{20} \\ &= \$12\ 828.54 \end{aligned}$$

c) What is the value and meaning of the function at $n = 0$?

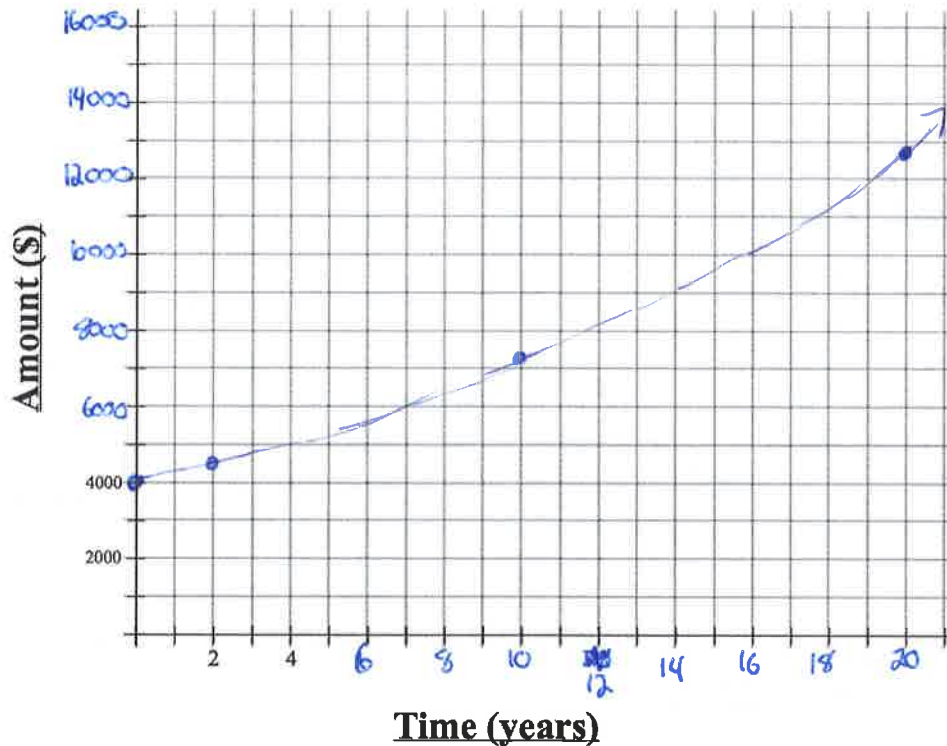
$$A = 4000(1.06)^0$$

$$A = 4000(1)$$

$$A = \$4000$$

This is the initial value.

d) Graph the relation for $n = 0$ to $n = 20$.



e) How long does it take for the investment to be worth \$20 000?

$$20000 = 4000 (1.06)^n$$

$$5 = 1.06^n$$

$$\log 5 = \log (1.06)^n$$

$$\log 5 = n \cdot \log 1.06$$

$$\frac{\log 5}{\log 1.06} = n$$

$$n = 27.62 \text{ years.}$$

5) A colony of bacteria starts with 400 cells and doubles in number every two days. A second colony of bacteria starts with 200 cells and quadruples in number every day. Write an equation to model the population of each colony, A , to time, t , in days.

Colony 1

$$A = 400(2)^{t/2}$$

Colony 2

$$A = 200(4)^t$$

BONUS: Bacteria A has an initial population of 500 and doubles every day, while bacteria B has an initial population of 50 and triples daily. After how long will the population of B overtake the population of A? What will their populations be at this point?

(Hint: product rule for logarithms)

$$500(2)^x = 50(3)^x$$

$$10(2)^x = 3^x$$

$$\log[10(2)^x] = \log 3^x$$

$$\log 10 + \log 2^x = \log 3^x$$

$$1 + x \cdot \log 2 = x \cdot \log 3$$

$$1 = x \cdot \log 3 - x \cdot \log 2$$

$$1 = x(\log 3 - \log 2)$$

$$\frac{1}{\log 3 - \log 2} = x$$

$$x = 5.679 \text{ days}$$

$$y = 25614.23$$

Answers

1) a)

Day	Population	First Differences	Second Differences
0	20	60	
1	80	240	180
2	320	960	720
3	1 280	3 840	2 880
4	5 120	15 360	11 520
5	20 480		

b) the values in each difference column increase by a factor of 4.

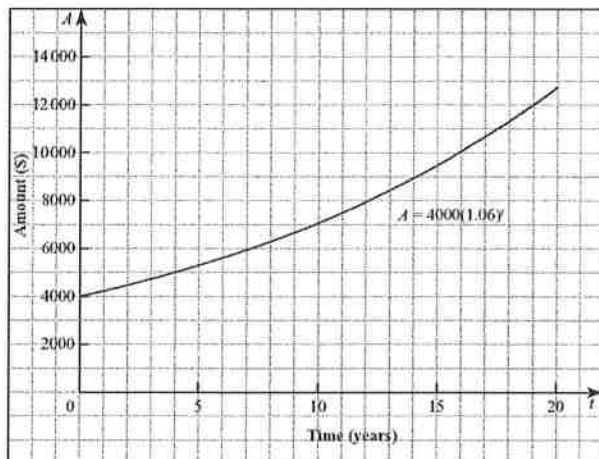
2) i) 10 ii) 20

3) a) $p = 200(3)^t$ b) 961 c) 106 288 200

4) a) $A = 4000(1.06)^n$ b) i) \$4494.40 ii) \$7163.39 iii) \$12 828.54

c) At $n = 0$, $A = \$4000$, which is the initial value of the investment.

d)



e) approximately 27.6 years

5) $A = 400(2)^{\frac{t}{2}}$ and $A = 200(4)^t$