

CHAPTER 6 - QUADRATIC EQUATIONS**6.1 - Completing the Square (Max and Min)
Part 1****Objective:** Go from standard form to vertex form

$$y = ax^2 + bx + c \longrightarrow y = a(x-h)^2 + k$$

Remember what a perfect square trinomial is!!!!

$$a^2 + 2ab + b^2 = (a+b)^2$$

or

$$a^2 - 2ab + b^2 = (a-b)^2$$

DO IT NOW!

Factor the following:

Rule: $(a+b)^2 = a^2 + 2ab + b^2$

Rule: $(a-b)^2 = a^2 - 2ab + b^2$

1) $x^2 + 4x + 4$

$$= (x+2)(x+2)$$

2) $x^2 + 18x + 81$

$$= (x+9)^2$$

$$(x+9)^2$$

3) $x^2 - 12x + 36$

$$(x-6)^2$$

Notice: The last term of a perfect square trinomial is half of the middle term squared!!!

and

the factored form is $(x + b/2)^2$

Fill in the blank to make this a perfect square trinomial:

$$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 3^2 = 9$$

$$x^2 + 6x + \underline{9}$$

To make this a perfect square trinomial we added $(b/2)^2$

$$\text{Factored form: } \left(x + \frac{6}{2}\right)^2$$

$$= (x+3)^2$$

To go from standard form to vertex form, you must go through the process of completing the square

Steps to Completing the Square:

- 1) Put brackets around the first two terms
- 2) Factor out the common number (not the letter)
- 3) Look at the last term in the brackets, divide it by 2 and then square it.
- 4) Add and subtract that term behind the last term in the brackets.
- 5) Move the negative term outside of the brackets by first multiplying it by the 'a' value.
- 6) Simplify the terms outside of the brackets.
- 7) Factor the perfect square trinomial

1a Convert the following equation into the vertex form (completing the square)

$$y = x^2 + 8x + 5$$

$$y = (x^2 + 8x) + 5$$

$$y = (x^2 + 8x + ?) + 5$$

$$\left(\frac{b}{a}\right)^2 = \left(\frac{8}{1}\right)^2 = 4^2 = 16$$

$$y = (x^2 + 8x + 16 - 16) + 5$$

$$y = 1(x^2 + 8x + 16 - 16) + 5$$

$$y = 1(x^2 + 8x + 16) - 16 + 5$$

$$y = 1\left(x + \frac{8}{2}\right)^2 - 11$$

$$y = 1(x + 4)^2 - 11$$

Steps to Completing the Square:

- 1) Put brackets around the first two terms
- 2) Factor out the common number (not the letter)
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- 4) Add and subtract that term behind the last term in the brackets.
- 5) Move the negative term outside of the brackets by first multiplying it by the 'a' value.
- 6) Simplify the terms outside of the brackets.
- 7) Factor the perfect square trinomial

b What is the vertex and axis of symmetry of the parabola? Is the vertex a max or min point?

Remember: for $y = a(x-h)^2 + k$:

The vertex is: (h, k)

The axis of symmetry is: $x = h$



vertex: $(-4, -11)$ is the min point.
a.o.s: $x = -4$

2a Convert the following equation into the vertex form (completing the square)

$$y = x^2 + 6x + 3$$

$$y = (x^2 + 6x) + 3$$

$$y = (x^2 + 6x + ?) + 3$$

$$\left(\frac{b}{a}\right)^2 = \left(\frac{6}{1}\right)^2 = 3^2 = 9$$

$$y = (x^2 + 6x + 9 - 9) + 3$$

$$y = (x^2 + 6x + 9) - 9 + 3$$

$$y = \left(x + \frac{6}{2}\right)^2 - 6$$

$$y = (x + 3)^2 - 6$$

Steps to Completing the Square:

- 1) Put brackets around the first two terms
- 2) Factor out the common number (not the letter)
- 3) Look at the last term in the brackets, divide it by 2 and then square it.
- 4) Add and subtract that term behind the last term in the brackets.
- 5) Move the negative term outside of the brackets by first multiplying it by the 'a' value.
- 6) Simplify the terms outside of the brackets.
- 7) Factor the perfect square trinomial

b What is the vertex and axis of symmetry of the parabola? Is the vertex a max or min point?

$(-3, -6)$ is the min point

a.o.s. $x = -3$

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TRY ON YOUR OWN!

Convert the following equation into the vertex form (completing the square). Then state the vertex and whether it is a max or min point.

$$y = x^2 + 4x + 5$$

$$y = (x^2 + 4x) + 5$$

$$\left(\frac{4}{2}\right) = 2^2 = 4$$

$$y = (x^2 + 4x + 4 - 4) + 5$$

$$y = (x^2 + 4x + 4) - 4 + 5$$

$$y = \left(x + \frac{4}{2}\right)^2 + 1$$

$$y = (x + 2)^2 + 1$$

vert: $(-2, 1)$ min

4a

Convert the following equation into the vertex form (completing the square)

$$y = x^2 - 12x + 8 \quad \text{'b' value is negative}$$

$$y = (x^2 - 12x) + 8$$

$$\left(\frac{-12}{2}\right) = (-6)^2 = 36$$

$$y = (x^2 - 12x + 36 - 36) + 8$$

$$y = (x^2 - 12x + 36) - 36 + 8$$

$$y = \left(x - \frac{12}{2}\right)^2 - 28$$

$$y = (x - 6)^2 - 28$$

b What is the vertex and axis of symmetry of the parabola? Is the vertex a max or min point?

$(6, -28)$ min point

a.o.s. $x = 6$

5a Convert the following equation into the vertex form (completing the square)

$$y = 2x^2 + 12x + 11$$

$$y = \left(\frac{2x^2}{2} + \frac{12x}{2} \right) + 11$$

$$y = 2(x^2 + 6x) + 11$$

$$\left(\frac{6}{2} \right)^2 = \left(\frac{6}{2} \right)^2 = 3^2 = 9$$

$$y = 2(x^2 + 6x + 9 - 9) + 11$$

$$y = 2(x^2 + 6x + 9) - 18 + 11$$

$$y = 2\left(x + \frac{6}{2}\right)^2 - 7$$

$$y = 2(x + 3)^2 - 7$$

Steps to Completing the Square:

- 1) Put brackets around the first two terms
- 2) Factor out the common number (not the letter)
- 3) Look at the last term in the brackets, divide it by 2 and then square it.
- 4) Add and subtract that term behind the last term in the brackets.
- 5) Move the negative term outside of the brackets by first multiplying it by the 'a' value.
- 6) Simplify the terms outside of the brackets.
- 7) Factor the perfect square trinomial

b What is the vertex and axis of symmetry of the parabola? Is the vertex a max or min point?

vertex: $(-3, -7)$ min point

a.o.s. $x = -3$

6a Convert the following equation into the vertex form (completing the square)

$$y = -x^2 - 6x + 4$$

$$y = (-x^2 - 6x) + 4$$

$$y = -1(x^2 + 6x) + 4$$

$$\left(\frac{6}{2} \right)^2 = \left(\frac{6}{2} \right)^2 = 3^2 = 9$$

$$y = -1(x^2 + 6x + 9 - 9) + 4$$

$$y = -1(x^2 + 6x + 9) + 9 + 4$$

$$y = -1\left(x + \frac{6}{2}\right)^2 + 13$$

$$y = -1(x + 3)^2 + 13$$

Steps to Completing the Square:

- 1) Put brackets around the first two terms
- 2) Factor out the common number (not the letter)
- 3) Look at the last term in the brackets, divide it by 2 and then square it.
- 4) Add and subtract that term behind the last term in the brackets.
- 5) Move the negative term outside of the brackets by first multiplying it by the 'a' value.
- 6) Simplify the terms outside of the brackets.
- 7) Factor the perfect square trinomial

b What is the vertex and axis of symmetry of the parabola? Is the vertex a max or min point?

vert: $(-3, 13)$ is the max point

a.o.s. $x = -3$

Lesson Summary:

- you can rewrite a quadratic relation of form $y = ax^2 + bx + c$ in the form $y = a(x-h)^2 + k$ by completing the square.

- for a quadratic relation in the form $y = a(x-h)^2 + k$, the vertex, (h, k) , represents the max or min point of the parabola. If $a > 0$, the vertex is a minimum point. If $a < 0$, the vertex is a maximum point.

- completing the square is used to find the maximum or minimum point of a quadratic relation that is in standard form.

Classwork / Homework: Pg. 270 # 3aceg, 4, 5, 7ace, 8ace