6.1 Completing the Square
MPM2D
Jensen

2. For each quadratic that is in standard form, determine the value of 'c' that makes each expression a perfect square trinomial (remember, the 'c' value is half of the 'b' value squared)

a) $x^2 + 6x + c$

9

c) $x^2 - 12x + c$

36

e) $x^2 + 2x + c$

1

3. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square

a) $y = x^2 + 6x - 1$

$y = (x^2 + 6x) - 1$

$y = (x^2 + 6x + 9 - 9) - 1$

$y = (x^2 + 6x + 9) - 9 - 1$

$y = (x + 3)^2 - 10$

c) $y = x^2 + 10x + 20$

$y = (x^2 + 10x) + 20$

$y = (x^2 + 10x + 25 - 25) + 20$

$y = (x^2 + 10x + 25) - 25 + 20$

$y = (x + 5)^2 - 5$

e) $y = x^2 - 6x - 4$

$y = (x^2 - 6x) - 4$

$y = (x^2 - 6x + 9 - 9) - 4$

$y = (x^2 - 6x + 9) - 9 - 4$

$y = (x - 3)^2 - 13$

g) $y = x^2 - 12x + 8$

$y = (x^2 - 12x) + 8$

$y = (x^2 - 12x + 36 - 36) + 8$

$y = (x^2 - 12x + 36) - 36 + 8$

$y = (x - 6)^2 - 28$

4. Find the vertex of each quadratic relation by completing the square

a) $y = x^2 + 6x + 2$

$y = (x^2 + 6x) + 2$

$y = (x^2 + 6x + 9 - 9) + 2$

$y = (x^2 + 6x + 9) - 9 + 2$

$y = (x + 3)^2 - 7$

Vertex : $(-3, -7)$

b) $y = x^2 + 12x + 30$

$y = (x^2 + 12x) + 30$

$y = (x^2 + 12x + 36 - 36) + 30$

$y = (x^2 + 12x + 36) - 36 + 30$

$y = (x + 6)^2 - 6$

vertex: $(-6, -6)$

c) $y = x^2 - 8x + 13$

$y = (x^2 - 8x) + 13$

$y = (x^2 - 8x + 16 - 16) + 13$

$y = (x^2 - 8x + 16) - 16 + 13$

$y = (x - 4)^2 - 3$

vertex : $(4, -3)$

d) $y = x^2 - 6x + 17$

$y = (x^2 - 6x) + 17$

$y = (x^2 - 6x + 9 - 9) + 17$

$y = (x^2 - 6x + 9) - 9 + 17$

$y = (x - 3)^2 + 8$

vertex: $(3, 8)$

7. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square

a) $y = -x^2 + 80x - 100$

$y = (-x^2 + 80x) - 100$

$y = -(x^2 - 80x) - 100$

$y = -(x^2 - 80x + 1600 - 1600) - 100$

$y = -(x^2 - 80x + 1600) + 1600 - 100$

$y = -(x - 40)^2 + 1500$

c) $y = 3x^2 + 90x + 50$

$y = (3x^2 + 90x) + 50$

$y = 3(x^2 + 30x) + 50$

$y = 3(x^2 + 30x + 225 - 225) + 50$

$y = 3(x^2 + 30x + 225) - 675 + 50$

$y = 3(x + 15)^2 - 625$

e) $y = -7x^2 + 14x - 3$

$y = (-7x^2 + 14x) - 3$

$y = -7(x^2 - 2x) - 3$

$y = -7(x^2 - 2x + 1 - 1) - 3$

$y = -7(x^2 - 2x + 1) + 7 - 3$

$y = -7(x - 1)^2 + 4$

8. Find the maximum or minimum point of each parabola by completing the square.

a) $y = -x^2 - 10x - 9$

$y = (-x^2 - 10x) - 9$

$y = -(x^2 + 10x) - 9$

$y = -(x^2 + 10x + 25 - 25) - 9$

$y = -(x^2 + 10x + 25) + 25 - 9$

$y = -(x + 5)^2 + 16$

max at $(-5, 16)$

c) $y = 2x^2 + 120x + 75$

$y = (2x^2 + 120x) + 75$

$y = 2(x^2 + 60x) + 75$

$y = 2(x^2 + 60x + 900 - 900) + 75$

$y = 2(x^2 + 60x + 900) - 1800 + 75$

$y = 2(x + 30)^2 - 1725$

min at $(-30, -1725)$

e) $y = -5x^2 - 200x - 120$

$y = (-5x^2 - 200x) - 120$

$y = -5(x^2 + 40x) - 120$

$y = -5(x^2 + 40x + 400 - 400) - 120$

$y = -5(x^2 + 40x + 400) + 2000 - 120$

$y = -5(x + 20)^2 + 1880$

max at $(-20, 1880)$

12. The path of a ball is modeled by the equation $y = -x^2 + 4x + 1$, where x is the horizontal distance, in meters, travelled and y is the height, in meters, of the ball above the ground. What is the maximum height of the ball, and at what horizontal distance does it occur?

$y = -(x^2 - 4x) + 1$

$y = -(x^2 - 4x + 4) + 4 + 1$

$y = -(x - 2)^2 + 5$

vertex : $(2, 5)$

max height is 5m at a horizontal distance of 2 meters.

Answers
2) a) 9 c) 36 e) 1
3) a) $y = (x + 3)^2 - 10$ c) $y = (x + 5)^2 - 5$ e) $y = (x - 3)^2 - 13$ g) $y = (x - 6)^2 - 29$
4) a) $(-3, -7)$ b) $(-6, 6)$ c) $(4, -3)$ d) $(3, 8)$
7) a) $y = -(x - 40)^2 + 1500$ c) $y = 3(x + 15)^2 - 625$ e) $y = -7(x - 1)^2 + 4$
8) a) max at $(-5, 16)$ c) min at $(-30, -1725)$ e) max at $(-20, 1880)$
12) max height of 5m occurs at a horizontal distance of 2m