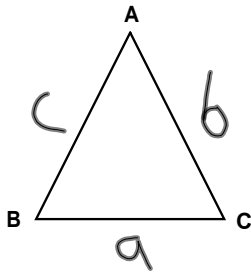


### Chapter 8: Trigonometry of Acute Triangles

#### 8.1 - The Sine Law



The relationship between the sides and their opposite angles in any acute  $\triangle ABC$  is:

#### Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

#### **DO IT NOW!!**

Solve for 'x':

1)  $\frac{1}{3} = \frac{x}{9}$

$$9 = \frac{3x}{3}$$

2)  $x = 3$

$$\frac{2}{6} = \frac{9}{x}$$

$$2x = \frac{54}{2}$$

$$x = 27$$

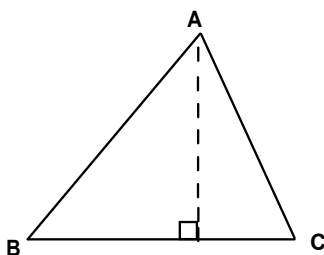
How can we use what we know about the trig ratios for right angle triangles to help us solve for unknown sides and angles of acute triangles?

#### Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine law is derived by breaking up an acute triangle into 2 right angle triangles and then using trig ratios (SOHCAHTOA).

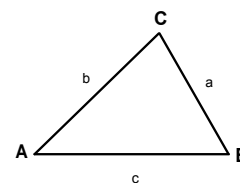
See the handout of the proof for the full explanation.



**Sine Law:** The ratio of each side to its opposite angle is equal!

#### Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

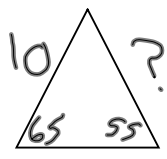


**Note:** Even though there are three parts to this equation, you only use two parts at a time. The choice of which two to use depends on what information is given.

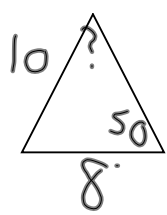
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**The Sine Law can be used to find:**

1. An unknown side when two angles and a side are known

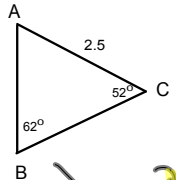


2. An unknown angle if two sides and the angle opposite one of the known sides are known



**Find Side Lengths Using the Sine Law**

1 Find the length of side ~~a~~ c



**Remember:** We can use the sine law to find an unknown side when we know two angles and a side.

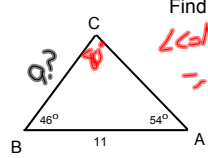
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{2.5}{\sin 62} = \frac{c}{\sin 52}$$

$$\frac{2.5(\sin 52)}{\sin 62} = \frac{c(\sin 62)}{\sin 62}$$

$$c = 2.2$$

2 Find the length of side 'a'



**Remember:** We can use the sine law to find an unknown side when we know two angles and a side.

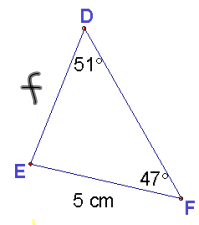
$$\frac{a}{\sin 54} = \frac{11}{\sin 46} = \frac{11}{\sin 80}$$

$$a(\sin 80) = 11(\sin 54)$$

$$a = \frac{11(\sin 54)}{\sin 80}$$

$$a = 9.0$$

3 Find the length of side 'f'



Remember: We can use the sine law to find an unknown side when we know two angles and a side.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{f}{\sin 47} = \frac{5}{\sin 51}$$

$$\frac{f}{\sin 47} = \frac{5}{\sin 51}$$

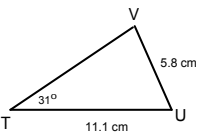
$$f(\sin 51) = 5(\sin 47)$$

$$f = \frac{5(\sin 47)}{\sin 51}$$

$$f = 4.7 \text{ cm}$$

**Find an Angle Using the Sine Law**

4 Find  $\angle V$



Remember: We can use the sine law to find an unknown angle if two sides and the angle opposite one of the known sides are known.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{5.8}{\sin 31} = \frac{11.1}{\sin V}$$

$$\frac{5.8}{\sin 31} = \frac{11.1}{\sin V}$$

$$5.8(\sin V) = 11.1(\sin 31)$$

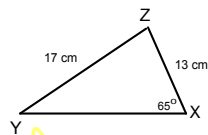
$$\sin V = \frac{11.1(\sin 31)}{5.8}$$

$$\sin V = 0.9856\dots$$

$$\angle V = \sin^{-1}(\text{answer})$$

$$\angle V = 80.3^\circ$$

5 Find  $\angle Y$



Remember: We can use the sine law to find an unknown angle if two sides and the angle opposite one of the known sides are known.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{17}{\sin 65} = \frac{13}{\sin Y}$$

$$\frac{17}{\sin 65} = \frac{13}{\sin Y}$$

$$17(\sin Y) = 13(\sin 65)$$

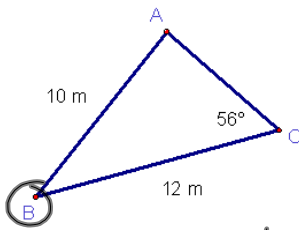
$$\sin Y = \frac{13(\sin 65)}{17}$$

$$\sin Y = 0.69\dots$$

$$\angle Y = \sin^{-1}(\text{answer})$$

$$\angle Y = 43.9^\circ$$

6

Find  $\angle B$ 

Remember: We can use the sine law to find an unknown angle if two sides and the angle opposite one of the known sides are known

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{12}{\sin A} = \frac{10}{\sin 56} = \frac{10}{\sin 56}$$

STEP 1:  $\angle A$

$$\frac{12}{\sin A} = \frac{10}{\sin 56}$$

$$\angle A = 84.2^\circ$$

## Homework: Pg. 402 #1-7,9

## Key Concepts

- In an acute  $\triangle ABC$ , the sine law states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- The sine law can be used to find

- an unknown side if two angles and a side are known
- an unknown angle if two sides and the angle opposite one of the known sides are known

- The sine law can also be written in the form

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

