

8.4/8.5 Volume and Surface Area of Cones

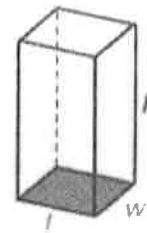
Part A: Review of what we know so far:

Volume is the measure of how much space a three dimensional object occupies.

The surface area of an object is the sum of the areas of its outside surfaces.
(How much material is needed to build an object)

Volume of a prism = area of base x height

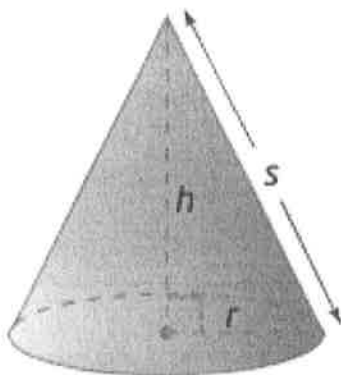
Volume of pyramid = $\frac{1}{3}$ (area base)(height)



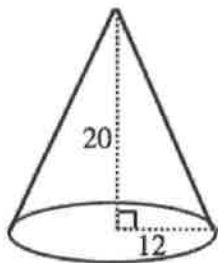
Part B: Volume of a Cone

The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height; therefore the formula for the volume of a cone is:

$$\text{Volume}_{\text{cone}} = \frac{1}{3} \pi r^2 h \quad \text{or} \quad \frac{\pi r^2 h}{3}$$



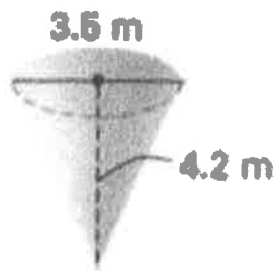
Example 1: Find the volume of the following cone:



$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (12^2)(20) \\ &= 3015.9 \end{aligned}$$

Volume_{cone} = 3015.9 units³

Example 2: Find the volume of the following cone:



$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (1.75)^2 (4.2) \\ &= 13.5 \text{ m}^3 \end{aligned}$$

Remember:

$$d = 2r$$

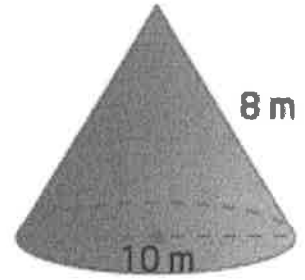
$$r = 1.75 \text{ m}$$

Volume_{cone} = 13.5 m³

Math 9

Jensen

Example 3: A conical pile of sand has a base diameter of 10 meters and a slant height of 8 meters. Determine the volume of the sand pile, to the nearest cubic meter.



Since the diameter of the base is 10m, the radius is 5.

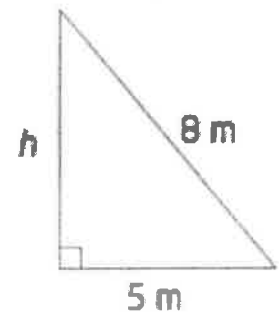
To determine the volume of the cone, we need to determine the **height**. Apply the Pythagorean theorem:

Height:

$$h^2 = 8^2 - 5^2$$

$$h^2 = 39$$

$$h = 6.2 \text{ m.}$$



Volume:

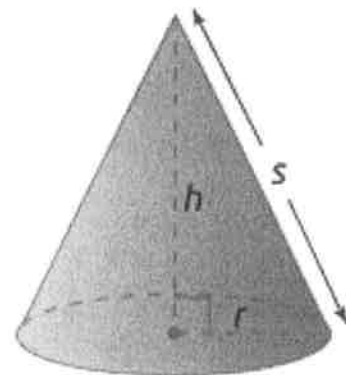
$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (5)^2 (6.2)$$

$$= 162.3 \text{ m}^3$$

Part C: Surface Area of a Cone

$$\text{Surface Area}_{\text{cone}} = \pi r s + \pi r^2$$



Example 4: Calculate the surface area of the cone, to the nearest square centimeter.

Start by finding the slant height by using the Pythagorean Theorem:

$$s^2 = 8^2 + 3^2$$

$$s^2 = 73$$

$$s = 8.5$$

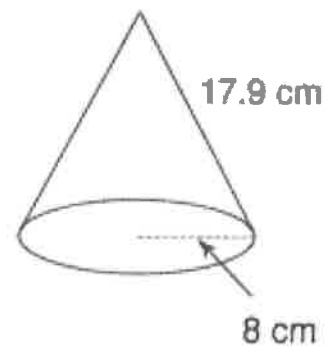


Now find the surface area of the cone:

$$\begin{aligned} SA &= \pi r s + \pi r^2 \\ &= \pi(3)(8.5) + \pi(3)^2 \\ &= 108.4 \text{ cm}^2 \end{aligned}$$

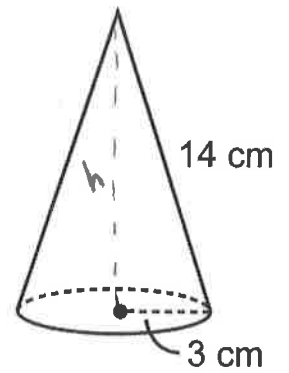
Example 5: Calculate the surface area of the cone, to the nearest square centimeter.

$$\begin{aligned} SA &= \pi r s + \pi r^2 \\ &= \pi(8)(17.9) + \pi(8)^2 \\ &= 650.9 \text{ cm}^2 \end{aligned}$$



Part D: Show me what you've learned

Find the volume and surface area of the following cone:



$$h^2 = 14^2 - 3^2$$

$$h^2 = 187$$

$$h = 13.7 \text{ cm}$$

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (3)^2 (13.7) \\ &= 129.1 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} SA &= \pi r s + \pi r^2 \\ &= \pi (3)(14) + \pi (3)^2 \\ &= 131.9 + 28.3 \\ &= 160.2 \text{ cm}^2 \end{aligned}$$

Volume = 129.1 cm³

Surface Area = 160.2 cm²