Calculus Midterm Exam Review

1) Differentiate each of the following with respect to x.

a) $y = 4$

b) $y = 3x$

c) $y = 5x^3 + 3x - 7$

d) $y = \frac{3x^2}{2}$

e) $y = \frac{3}{x^4}$

f) $y = 4\sqrt[3]{x} - 8$

g) $y = (2x + 3)(5x - 1)$

h) $y = (4x^2 - x + 2)(x - 3)$

i) $y = (2x + 1)(x^2 - 3)(3x - 2)$

j) $y = (3x - 2)^2$

k) $y = \sqrt{2x - 5}$

l) $y = \frac{1}{(4x^2 - 3x)^3}$

m) $y = (x^2 + 2x)^3(4x + 1)^2$

n) $y = \frac{2x}{x-5}$

o) $y = \frac{3x}{x^2+2x-4}$

p) $y = \frac{3x}{(x-2)^2}$

q) $y = \left[\frac{1}{(4x+x^2)^3}\right]^3$

r) $y = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{3}} + 6\sqrt[3]{x}$

2) Determine $f''(-1)$ for the function $f(x) = \frac{3}{(2x-3)^2}$

3) Determine the equation of the tangent to the curve $y = (2x^2 - 3)(x + 1)^2$ at the point where $x = 1$.

4) For what values of x does the curve $y = -x^3 + 6x^2$ have a tangent with a slope of 12 ?

5) A tangent to the parabola $y = 3x^2 - 7x + 5$ is perpendicular to $x + 5y - 10 = 0$. Determine the equation of the tangent.

6) Water is being drained from the bottom of a tank. The equation that gives the volume *V* of water, in liters, remaining in the tank after *t* minutes is $V(t) = 3000\left(1 - \frac{t}{10}\right)^2$, $0 \le t \le 10$.

a) Find the volume of water in the tank after 5 min.

b) Find the rate at which the water is draining from the tank after

i) 3 min

ii) 8 min

7) For the function $f(x) = x^3 - x^2 - x + 1$, determine the points where the tangent line is horizontal.

8) The amount of pollution in a certain lake is $P(t) = \left(t^{\frac{1}{4}} + 3\right)^3$, where t is measured in years and P is measured in parts per million (ppm). At what rate is the amount of pollution changing after 16 years?

9) An object moves in a straight line, and its position, s, in metres after t seconds is $s(t) = 8 - 7t - t^2$.

a) Determine the velocity when t=5

b) Determine the acceleration when t=5.

10) A store sells 250 pairs of Brand X running shoes each month when priced at $100 per pair. It has been determined that for every $5 decrease in price, an additional 20 pairs will be sold.

a) Determine the demand or price function.

b) Determine the revenue function.

c) Find the revenue when sales are 250 pairs and 290 pairs of shoes per month.

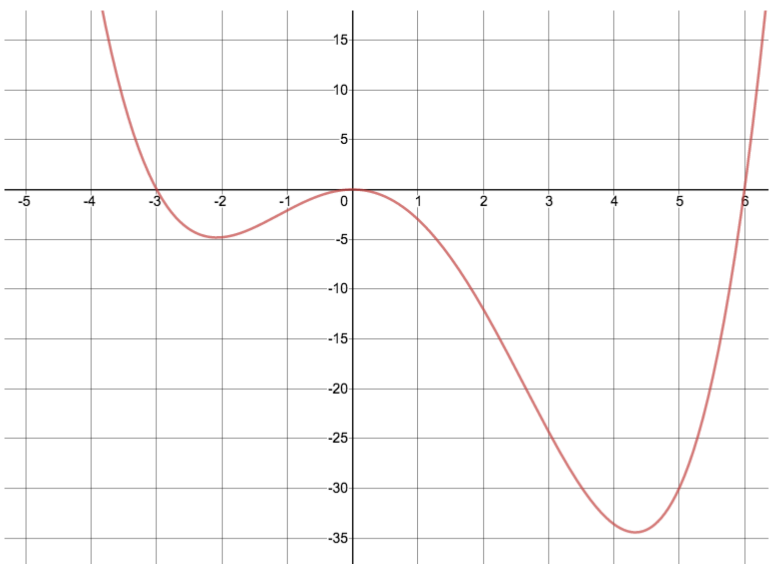
d) Determine the marginal revenue when sales are 290 pairs of shoes per month.

11) A theatre has found that if the price of a ticket is \$24 then 400 tickets will be sold. For every \$2 decrease in price, an additional 50 tickets will be sold.
a) Determine the demand or price function based on the number of price decreases.
b) Determine a revenue function based on the number of price decreases.
c) Determine the marginal revenue for the revenue function developed in part a).
d) When is this marginal revenue function equal to zero? What is the total revenue at this time? How can the owners of the theatre use this information?
e) If the theatre has a maximum capacity of 450, what price will maximize revenue?

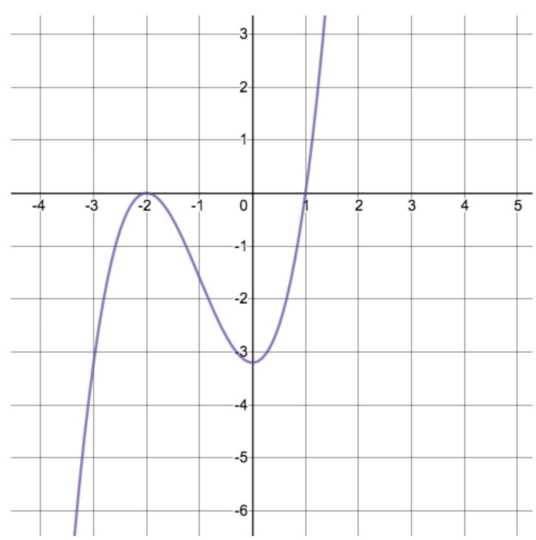
12) Given the revenue function $R(x) = 30x - 0.025x^2$ and the cost function $C(x) = 2x + 5$, where x is the number of items being sold, determine:
a) the revenue and marginal revenue when 500 items are sold
b) the cost and marginal cost when 500 items are produced
c) the profit and marginal profit from sales of 500 items

13) Determine the equation of all tangent lines to the curve $y = 2x - \frac{8}{x}$ that have a slope of 4.

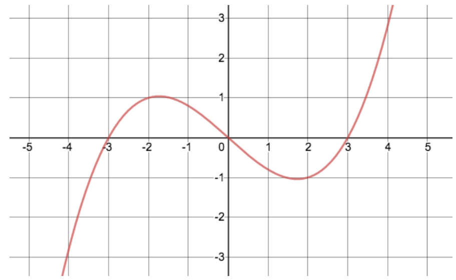
14) Sketch a graph of the first and second derivative for the function $f(x)$ shown:



15) Given the graph of $f'(x)$, state the intervals of increase and decrease for the function $f(x)$. Then sketch a possible graph of $f(x)$.



16) Given the graph of $f''(x)$, state the intervals of concavity for the function $f(x)$.



17) Find the absolute maximum and minimum values of $f(x) = x^3 + 3x^2 + 1$ on the interval $-2 \le x \le 2$.

18) Consider the function defined by $f(x) = -4x^3 + 6x^2 + 2$

a) Find the critical points and classify them using the second derivative test.

b) Find the points of inflection and identify the intervals of concavity.

19) Determine the local extrema for each function and classify them as local maxima or minima.

a) $y = 81 - x^4$

b) $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 9$

c) $f(x) = -3x^3 + 9x^2 - 8$

20) Determine the intervals over which each function is increasing/decreasing.

a) $y = x^3 - 12x + 8$

b) $f(x) = 3x^4 - 2x^3 - 9x^2$

21) Determine the points of inflection and intervals of concavity for each function.

a) $y = \frac{x^4}{3} - 8x^2$

b) $f(x) = x^4 - 8x^3 + 3x - 5$

22) Sketch each function using the algorithm for curve sketching.

a) $h(x) = x^3 + 6x^2 + 9x + 4$

b) $f(x) = -3x^4 - 2x^3 + 15x^2 - 12x + 2$

c) $f(x) = -x^4 - x^3 + 2x^2$

d) $y = \frac{5x}{x^2-1}$

e) $y = \frac{1-x^2}{x^2+1}$

f) $f(x) = 3x^4 - 16x^3 + 18x^2$

23) Iesha is making a jewelry box for her mother in technology class. The top and bottom of the box will be square and made out of a wood that costs \$0.002/cm^2, and the sides of the box will be made from a cheaper wood that costs \$0.001/cm^2. Find the dimensions of the box that will minimize the cost if she wants the box to have a volume of 4800 cm^3.

24) A manufacturing company is responsible for designing a 500 mL can for diced tomatoes.

a) Determine the dimensions of the can that will minimize the surface area of the can.

b) The lid and base of the can are made out of a stronger metal that costs twice as much as the sides. What dimensions will minimize the cost of the producing the can? How does your answer compare to your answer in part a)?

25) A farmer has 800 m of fencing to enclose two rectangular pens. The pens should have equal areas. What dimensions will maximize the enclosed area?

26) A piece of wire, 200 cm long, is to be bent into an isosceles triangle. What should the dimensions of the triangle be in order to maximize its area?

27) Differentiate each of the following with respect to x.

a) $y = 2x + 3e^x$
b) $y = 10^x$
c) $y = 4^{3x^2}$
d) $y = (x^4)2^x$
e) $3\sin x + \cos(2x)$
f) $y = \tan(3x)$
g) $y = [\sin(2x)]e^{3x}$
h) $y = x^2 \sin x$
i) $y = 5(2)^x$
j) $y = -2(10)^{3x}$
k) $y = \sin^3(x^2)$
l) $y = \tan\sqrt{1-x}$
m) $[\sin(2x) + 1]^4$
n) $\sin[\cos(x^2)]$
o) $y = 2\sin x - 3\cos(5x)$
p) $x^2y + y^2x = 1$
q) $x = \sin(y)$
r) $y^5 + xy = 3$
s) $\sin y + \sin x = 1$
t) $yx^2 + e^y = x$
u) $y = \ln(x^3 + 3x)$
v) $y = 4\log_5 x$
w) $y = \log_2(\cos x)$
x) $y = \log(4x^2)$
y) $y = x^2 e^{\sin x}$

28) Find the slope of the line tangent to the curve $y = \sin(\cos x)$ at $x = \frac{\pi}{2}$

29) Find the equation of the line tangent to the curve $y = 2x^2 + \sin(4x)$ at $x = \frac{\pi}{3}$

30) The function for a current is $I(t) = 9\cos(120\pi t)$ where *I* is the current, in amperes, and *t* is the time in seconds.
a) Find the current when *t* = 6 s.
b) The electromotive force, *E*(*t*), in a circuit, is given in volts by finding the first derivative of the current function *I*(*t*). Find the function for the electromotive force *E*(*t*).
c) Determine the electromotive force when *t* = 3.
d) Find the maximum and minimum electromotive forces and the times that they occur.

31) The movement of an engine piston can be modelled by $h(t) = 4\sin t$ where *h* is the height of the piston, in centimetres, above the neutral position and *t* is time, in seconds.
a) Determine the velocity of the piston when *t* = 5.
b) Determine the acceleration of the piston when *t* = 5.

32) A virus is spreading according to the function $p(t) = 125(2)^{\frac{t}{4}}$ where $p(t)$ is the number of people infected after t days.
a) How many people had the virus initially?
b) How many people will be infected after 4 weeks?
c) How fast will the virus be spreading after 4 weeks?
d) How long will it take until 22 500 people are infected?

33) Find the equation of the line tangent to the curve $y = \left(\frac{1}{2}\right)^x$ at $(-3,8)$

34) Determine the equation of the tangent to the curve $y = x\sin x$ at the point where $x = \frac{\pi}{2}$.

35) Determine all the maximum and minimum values of $y = xe^x + 3e^x$.

36) Determine the equation of the tangent to the curve defined by $y = 2e^{3x}$ that is parallel to the line defined by $-6x + y = 2$.

Answer Key

1)a) $y' = 0$ **b)** $y' = 3$ **c)** $y' = 15x^2 + 3$ **d)** $y' = 3x$ **e)** $y' = -\frac{12}{x^5}$ **f)** $y' = \frac{4}{3x^{\frac{2}{3}}}$ **g)** $y' = 20x + 13$

h) $y' = 12x^2 - 26x + 5$ **i)** $y' = 24x^3 - 3x^2 - 40x + 3$ **j)** $y' = 18x - 12$ **k)** $y' = \frac{1}{\sqrt{2x-5}}$ **l)** $y' = -\frac{3(8x-3)}{(4x^2-3x)^4}$

m) $y' = 2(4x+1)(x^2+2x)^2(16x^2+23x+3)$ **n)** $y' = -\frac{10}{(x-5)^2}$ **o)** $y' = -\frac{3(x^2+4)}{(x^2+2x-4)^2}$ **p)** $y' = -\frac{3(x+2)}{(x-2)^3}$

q) $y' = \frac{-9(2x+4)}{(4x+x^2)^{10}}$ **r)** $y' = \frac{2}{x^{\frac{2}{3}}} - \frac{1}{x^{\frac{3}{2}}} + \frac{1}{\sqrt{3}}$

2) $f''(-1) = \frac{72}{625}$ **3)** $y = 12x - 16$ **4)** $x = 2$ **5)** $y = 5x - 7$ **6)a)** 750 L **b)i)** -420 L/min **ii)** -120 L/min

7) $(-\frac{1}{3}, 1.185)$ and $(1,0)$ **8)** 2.34 ppm/year **9)a)** -17 m/s **b)** -2 m/s^2

10)a) $p(x) = -\frac{1}{4}x + 162.5$ **b)** $R(x) = -\frac{1}{4}x^2 + 162.5x$ **c)** R(250)= \$25,000 R(290)= \$26,100

d) \$17.50/pair of shoes sold

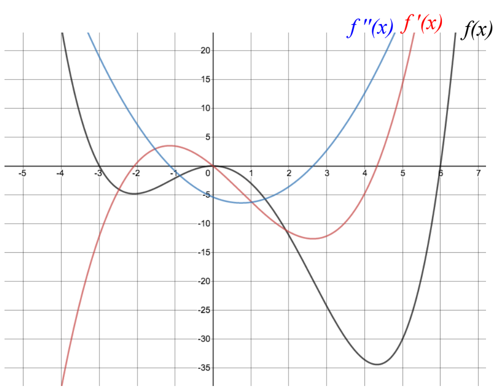
11)a) p(x) = 24 – 2x **b)** R(x) = (24 – 2x)(400 + 50x) **c)** R'(x)= 400 – 200x **d)** when the price is \$20, revenue is \$10 000, the owners can use this information to choose the best price to charge for tickets e) \$22

12)a) $R(500) = \$8750 \quad and \quad R'(500) = \frac{\$5}{item}$

b) $C(500) = \$1005 \quad and \quad C'(500) = \frac{\$2}{item}$ **c)** $P(500) = \$7745 \quad and \quad P'(500) = \$3/item$

13) $y = 4x + 8 \quad and \quad y = 4x - 8$

14)



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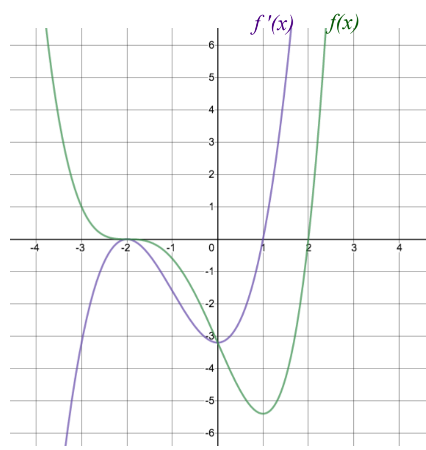


Increasing: $x > 1$
Decreasing: $x < -2, -2 < x < 1$

16) concave up: $-3 < x < 0, x > 3$
concave down: $x < -3, 0 < x < 3$

17) absolute min: (0,1)
absolute max: (2,21)

18) local min: (0,2)
local max: (1,4)
points of inflection: (0.5,3)
concave up: $x < 0.5$
concave down: $x > 0.5$

19)a) local max: (0,81) **b)** local min:(-2,17) and (1,-10)
local max: (-1,22) **c)** local min: (0,-8)
local max: (2,4)

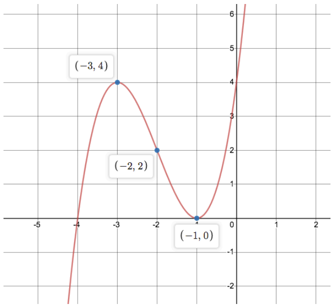
20)a) increasing: $x < -2, x > 2$
decreasing: $-2 < x < 2$

b) increasing: $-1 < x < 0$ and $x > 1.5$
decreasing: $x < -1$ and $0 < x < 1.5$

21)a) points of inflection: $\left(-2, -\frac{80}{3}\right)$ and $\left(2, -\frac{80}{3}\right)$
concave down: $-2 < x < 2$
concave up: $x < -2, x > 2$

b) points of inflection: (0,-5) and (4, -249)
concave down: $0 < x < 4$
concave up: $x < 0, x > 4$

22)a)



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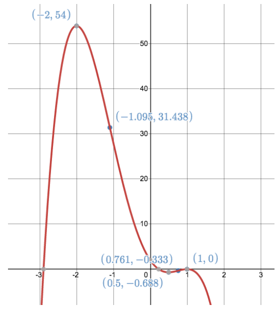

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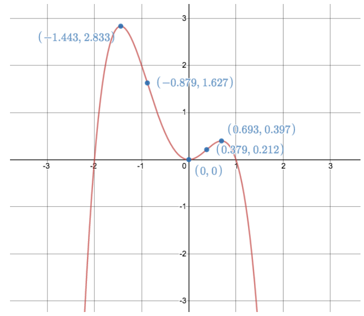

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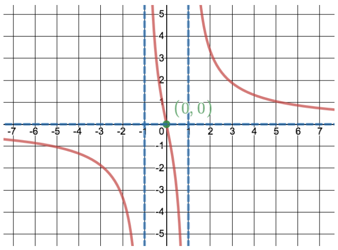

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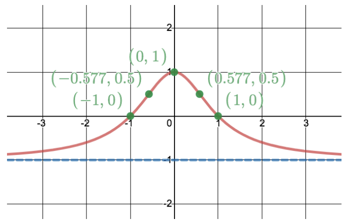

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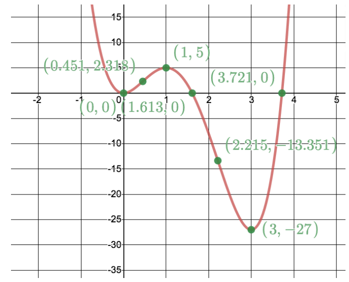

23) 13.4cm x 13.4cm x 27.8cm

24)a) r=4.3cm and h=8.6cm **b)** r=3.41cm and h=13.66cm

25) 3 parallel fences of 133.33 m by 2 parallel fences of 100 m

26) All sides are $\frac{200}{3}$ cm

27)a) $y' = 2 + 3e^x$ **b)** $y' = 10^x \ln 10$ **c)** $y' = 6x(4)^{3x^2}\ln(4)$ **d)** $y' = x^3 2^x[4 + x\ln(2)]$
e) $y' = 3\cos x - 2\sin(2x)$ **f)** $y' = 3\sec^2(3x)$ **g)** $y' = e^{3x}[3\sin(2x) + 2\cos(2x)]$ **h)** $y' = x(2\sin x + x\cos x)$
i) $y' = 5(\ln 2)(2)^x$ **j)** $y' = -6[\ln(10)](10)^{3x}$ **k)** $y' = 6x\cos(x^2)\sin^2(x^2)$ **l)** $y' = \frac{-\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$
m) $y' = 8\cos(2x)[\sin(2x) + 1]^3$ **n)** $y' = -2x\sin(x^2)\cos[\cos(x^2)]$ **o)** $y' = 2\cos x + 15\sin(5x)$
p) $y' = -\frac{y^2+2xy}{2xy+x^2}$ **q)** $y' = \frac{1}{\cos y}$ **r)** $y' = \frac{-y}{5y^4+x}$ **s)** $y' = -\frac{\cos x}{\cos y}$ **t)** $y' = \frac{1-2xy}{e^y+x^2}$ **u)** $y' = \frac{3x^2+3}{x^3+3x}$ **v)** $y' = \frac{4}{x\ln 5}$
w) $y' = \frac{-\sin x}{\cos x \ln 2}$ **x)** $y' = \frac{2}{x\ln 10}$ **y)** $y' = xe^{\sin x}(x\cos x + 2)$

28) $m = -1$ **29)** $y = \frac{4\pi-6}{3}x + \frac{12\pi-4\pi^2-9\sqrt{3}}{18}$ **30)a)** 9 A **b)** $E(t) = -1080\pi\sin(120\pi t)$ **c)** 0 V
d) min is -3392.9 V when $t = \frac{1+4k}{240}$ $where\ k\epsilon\mathbb{Z}, k \geq 0$ and max is 3392.9 V when $t = \frac{3+4k}{240}$ $and\ k\epsilon\mathbb{Z}\ and\ k \geq 0$

31)a) $1.135 cm/s$ **b)** $3.836 cm/s^2$

32)a) 125 people **b)** 16000 people **c)** 2773 people/day **d)** about 30 days

33) $y = 8\ln\left(\frac{1}{2}\right)x + 8 + 24\ln\left(\frac{1}{2}\right)$

34) $y = x$

35) Local min at $(-4, -e^{-4})$
No local max

36) $y = 6x + 2$