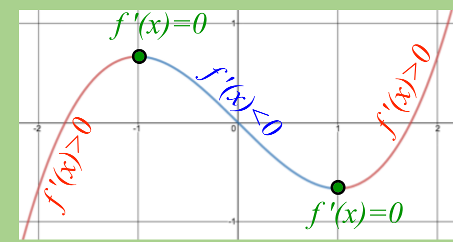
Derivative Rules

Rule	Derivative
Power Rule If $f(x) = x^n$	$f'(x) = nx^{n-1}$
Constant Multiple Rule If $f(x) = c \cdot g(x)$ where c is a constant	$f'(x) = c \cdot g'(x)$
Sum Rule If $h(x) = f(x) + g(x)$	$h'(x) = f'(x) + g'(x)$
Difference Rule If $h(x) = f(x) - g(x)$	$h'(x) = f'(x) - g'(x)$
Product Rule If $h(x) = f(x)g(x)$	$h'(x) = f'(x)g(x) + g'(x)f(x)$
Quotient Rule If $h(x) = f(x) \div g(x)$	$h'(x) = \dfrac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$
Power of a Function Rule If $h(x) = (f(x))^n$	$h'(x) = n[f(x)]^{n-1} \times f'(x)$
Chain Rule If $h(x) = f(g(x))$	$h'(x) = f'[g(x)] \times g'(x)$
Exponential Functions If $f(x) = b^x$ $g(x) = e^x$	$f'(x) = b^x \times \ln b$ $g'(x) = e^x$
Trig Functions If $f(x) = \sin x$ $g(x) = \cos x$ $h(x) = \tan x$	$f'(x) = \cos x$ $g'(x) = -\sin x$ $h'(x) = \sec^2 x$
Log Functions If $g(x) = \log_a[x]$ $h(x) = \ln x$	$g'(x) = \dfrac{1}{x \ln a}$ $h'(x) = \dfrac{1}{x}$

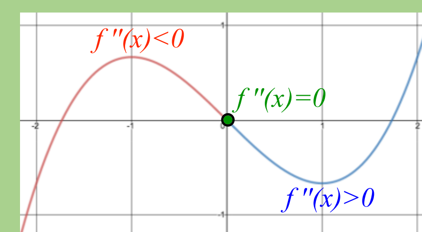
Relationship between $f(x)$, $f'(x)$, and $f''(x)$

In each diagram, you are shown the graph of $f(x)$ and given information about either $f'(x)$ or $f''(x)$

$f'(x) = 0$	Horizontal tangent; possible local extrema (turning point)
$f'(x) > 0$	$f(x)$ is increasing
$f'(x) < 0$	$f(x)$ is decreasing



$f''(x) = 0$	Possible point of inflection (change in concavity)
$f''(x) > 0$	$f(x)$ is concave up
$f''(x) < 0$	$f(x)$ is concave down



Tests of Critical Numbers:

Absolute Extrema on an Interval $[a, b]$	1. Find CN $x = c$, at $f'(x) = 0$ or undefined 2. Check endpoints and critical numbers; $f(a), f(c), f(b)$ 3. Choose the minimum and maximum values
Local Extrema – First Derivative Test of Critical Numbers	1. Find CN $x = c$, at $f'(x) = 0$ or undefined 2. Make a sign chart for $f'(x)$. Use test values. 3. Draw conclusions about $f(x)$ - If $f(x)$ changes from increasing to decreasing, $(c, f(c))$ is a local max - If $f(x)$ changes from decreasing to increasing, $(c, f(c))$ is a local min
Local Extrema – Second Derivative Test of Critical Numbers	1. Find CN $x = c$, at $f'(x) = 0$ or undefined 2. Calculate the second derivative $f''(x)$ 3. Test the critical numbers in $f''(x)$ - if $f''(c) > 0$, $f(x)$ is concave up at $(c, f(c))$ and it is a local min - if $f''(c) < 0$, $f(x)$ is concave down at $(c, f(c))$ and it is a local max - if $f''(c) = 0$, the test fails and you must use the First Derivative Test