

Intro to Rational Expressions - Fractions and Exponents Review - Worksheet

MCR3U

Jensen

SOLUTIONS

1) Add or subtract the following fractions.

$$\text{a) } \frac{2}{3} + \frac{3}{4}$$

$$= \frac{8}{12} + \frac{9}{12}$$

$$= \frac{17}{12}$$

$$\text{b) } \frac{3}{5} - \frac{1}{3}$$

$$= \frac{9}{15} - \frac{5}{15}$$

$$= \frac{4}{15}$$

$$\text{c) } \frac{x}{3} + \frac{y}{2}$$

$$= \frac{2x}{6} + \frac{3y}{6}$$

$$= \frac{2x+3y}{6}$$

$$\text{d) } \frac{3x}{5} - \frac{2y}{7}$$

$$= \frac{21x}{35} - \frac{10y}{35}$$

$$= \frac{21x-10y}{35}$$

2) Simplify (multiplication)

$$\text{a) } \left(\frac{2}{3}\right)\left(-\frac{1}{5}\right)$$

$$= -\frac{2}{15}$$

$$\text{b) } \left(\frac{4}{5}\right)\left(\frac{25}{12}\right)$$

$$= \frac{5}{3}$$

$$\text{c) } \left(\frac{17}{8}\right)\left(\sqrt{\frac{13}{289}}\right)$$

$$= \frac{17}{8}\left(\frac{\sqrt{13}}{17}\right)$$

$$= \frac{\sqrt{13}}{8}$$

$$\text{d) } \left(-\frac{5}{8}\right)\left(\sqrt{\frac{75}{128}}\right)$$

$$= \left(-\frac{5}{8}\right)\left(\frac{5\sqrt{3}}{8\sqrt{2}}\right)$$

$$= -\frac{25\sqrt{3}}{64\sqrt{2}}$$

3) Simplify (multiplication and division)

$$\text{a) } \frac{4}{9} \cdot \frac{7}{4}$$

$$= \frac{7}{9}$$

$$\text{b) } \frac{-11}{5} \cdot \frac{-7}{4}$$

$$= \frac{77}{20}$$

$$\text{c) } \frac{-17}{10} \div \frac{9}{4}$$

$$= \frac{-17}{10} \times \frac{4}{9}$$

$$= -\frac{34}{45}$$

$$\text{d) } \frac{6}{7} \div \frac{7}{6}$$

$$= \frac{6}{7} \times \frac{6}{7}$$

$$= \frac{36}{49}$$

$$\text{e) } \frac{7}{3} \cdot 6$$

$$= 14$$

4) Simplify (exponents)

a) $(x^6)(x^4)$

$$= x^{10}$$

b) $(y^3)(y^7)(y^9)$

$$= y^{19}$$

c) $m^5 \div m^4$

$$= m$$

d) $h^6 \div h^8$

$$= h^{-2}$$

$$= \frac{1}{h^2}$$

e) $\sqrt{h^6} + \sqrt{h^8}$

$$= (h^6)^{\frac{1}{2}} + (h^8)^{\frac{1}{2}}$$

$$= h^3 + h^4$$

$$= \frac{1}{h}$$

f) $\frac{x^{10}}{x^5}$

$$= x^5$$

g) $(yz^2)^3$

$$= y^3 z^6$$

h) $[-(x)^2]^2$

$$= x^4$$

i) $(x^2)^3 (y^3)^2$

$$= x^6 y^6$$

j) $\frac{(-x)^2}{(-x)}$

$$= \frac{x^2}{-x^1}$$

$$= -x$$

5) Simplify fractions and exponents

a) $\frac{7x \cdot \cancel{12}y^4}{\cancel{8}y \cdot 5x^2} + \frac{2}{3}$

$$\frac{28xy}{5x^2y} + \frac{2}{3}$$

$$\frac{28}{5x} + \frac{2}{3}$$

$$\frac{84}{15x} + \frac{10x}{15x}$$

$$= \frac{84+10x}{15x}$$

b) $\frac{\cancel{5}25x^3 \cdot 7y^2}{3x \cdot \cancel{5}yx^2}$

$$= \frac{35x^3y^2}{3x^3y}$$

$$= \frac{35y}{3}$$

6) Simplify. Your answer should contain only positive exponents.

$$\begin{aligned} \text{a) } & (x^{-2}x^{-3})^4 \\ & = (x^{-5})^4 \\ & = x^{-20} \\ & = \frac{1}{x^{20}} \end{aligned}$$

$$\begin{aligned} \text{b) } & (x^4)^{-3} \cdot 2x^4 \\ & = x^{-12} \cdot 2x^4 \\ & = 2x^{-8} \\ & = \frac{2}{x^8} \end{aligned}$$

$$\begin{aligned} \text{c) } & (n^3)^3 \cdot 2n^{-1} \\ & = n^9 \cdot 2n^{-1} \\ & = 2n^8 \end{aligned}$$

$$\begin{aligned} \text{d) } & (2v)^2 \cdot 2v^2 \\ & = 4v^2 \cdot 2v^2 \\ & = 8v^4 \end{aligned}$$

$$\begin{aligned} \text{e) } & \frac{2x^2y^4 \cdot 4x^2y^4 \cdot 3x}{3x^{-3}y^2} \\ & = \frac{24x^5y^8}{3x^{-3}y^2} \\ & = 8x^8y^6 \end{aligned}$$

$$\begin{aligned} \text{f) } & \frac{2y^3 \cdot 3xy^3}{3x^2y^4} \\ & = \frac{6xy^6}{3x^2y^4} \\ & = \frac{2y^2}{x} \end{aligned}$$

$$\begin{aligned} \text{g) } & \frac{x^3y^3 \cdot x^3}{4x^2} \\ & = \frac{x^6y^3}{4x^2} \\ & = \frac{x^4y^3}{4} \end{aligned}$$

$$\begin{aligned} \text{h) } & \frac{3x^2y^2}{2x^{-1} \cdot 4yx^2} \\ & = \frac{3x^2y^2}{8xy} \\ & = \frac{3xy}{8} \end{aligned}$$

$$\begin{aligned} \text{i) } & \frac{x}{(2x^0)^2} \\ & = \frac{x}{4} \end{aligned}$$

$$\begin{aligned} \text{j) } & \frac{2m^{-4}}{(2m^{-4})^3} \\ & = \frac{2m^{-4}}{8m^{-12}} \\ & = \frac{m^8}{4} \end{aligned}$$

Answers

1. a) $\frac{17}{12}$ b) $\frac{4}{15}$ c) $\frac{2x+3y}{6}$ d) $\frac{21x-10y}{35}$

2. a) $-\frac{2}{15}$ b) $\frac{5}{3}$ c) $\frac{\sqrt{13}}{8}$ d) $-\frac{25\sqrt{3}}{64\sqrt{2}}$

3. a) $(7/9)$ b) $(77/20)$ c) $(-34/45)$ d) $(36/49)$ e) 14

4. a) x^{10} b) y^{19} c) m d) ~~$\frac{1}{h}$~~ e) $1/h$ f) x^5 g) y^3z^6 h) x^4 i) x^6y^6 j) $-x$

5. a) $\frac{84+10x}{15x}$ b) $\frac{35y}{3}$

6. a) $\frac{1}{x^{20}}$ b) $\frac{2}{x^8}$ c) $2n^8$ d) $8v^4$ e) $8x^8y^6$ f) $\frac{2y^2}{x}$ g) $\frac{x^4y^3}{4}$ h) $\frac{3xy}{8}$ i) $\frac{x}{4}$ j) $\frac{m^8}{4}$

3.3 Rational Exponents - Worksheet

MCR3U

Jensen

SOLUTIONS

1) Evaluate each cube root.

a) $\sqrt[3]{64}$

$$= 4$$

b) $(-1000)^{\frac{1}{3}}$

$$= -10$$

c) $\sqrt[3]{\frac{1}{8}}$

$$= \frac{1}{2}$$

d) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$$= \frac{2}{3}$$

2) Evaluate each root.

a) $81^{\frac{1}{4}}$

$$= 3$$

b) $\sqrt[4]{\frac{16}{625}}$

$$= \frac{2}{5}$$

c) $64^{\frac{1}{6}}$

$$= 2$$

d) $\sqrt[5]{-100\,000}$

$$= -10$$

3) Evaluate.

a) $8^{\frac{2}{3}}$

$$= \left(8^{\frac{1}{3}}\right)^2$$

$$= 4$$

b) $32^{\frac{4}{5}}$

$$= \left(32^{\frac{1}{5}}\right)^4$$

$$= 16$$

c) $(-64)^{\frac{5}{3}}$

$$= \left[(-64)^{\frac{1}{3}}\right]^5$$

$$= -1024$$

d) $\left(\frac{1}{10\,000}\right)^{\frac{3}{4}}$

$$= \frac{1}{1000}$$

4) Evaluate.

a) $16^{-\frac{1}{4}}$

$$= \frac{1}{16^{\frac{1}{4}}}$$

$$= \frac{1}{2}$$

b) $25^{-\frac{3}{2}}$

$$= \frac{1}{25^{\frac{3}{2}}}$$

$$= \frac{1}{125}$$

c) $\left(\frac{1}{8}\right)^{-\frac{7}{3}}$

$$= \left(\frac{8}{1}\right)^{\frac{7}{3}}$$

$$= 128$$

d) $\left(-\frac{1}{32}\right)^{-\frac{2}{5}}$

$$= \left(-\frac{32}{1}\right)^{\frac{2}{5}}$$

$$= 4$$

e) $\left(\frac{10\,000}{81}\right)^{-\frac{3}{4}}$

$$= \left(\frac{81}{10\,000}\right)^{\frac{3}{4}}$$

$$= \frac{27}{1000}$$

f) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$

$$= \left(-\frac{27}{8}\right)^{\frac{2}{3}}$$

$$= \frac{9}{4}$$

5) Simplify. Express your answers using only positive exponents.

$$\text{a) } x^{\frac{1}{4}} \cdot x^{\frac{1}{4}}$$

$$= x^{\frac{1}{2}}$$

$$\text{b) } (m^{\frac{1}{3}})(m^{\frac{3}{4}})$$

$$= m^{\frac{4}{12}} \cdot m^{\frac{9}{12}}$$

$$= m^{\frac{13}{12}}$$

$$\text{c) } \frac{w^{\frac{1}{2}}}{w^{\frac{1}{3}}} = \frac{w^{\frac{3}{6}}}{w^{\frac{2}{6}}}$$

$$= w^{\frac{1}{6}}$$

$$\text{d) } \frac{ab^2}{a^{\frac{1}{2}}b^{\frac{1}{3}}} = \frac{a^{\frac{2}{2}}b^{\frac{6}{3}}}{a^{\frac{1}{2}}b^{\frac{1}{3}}}$$

$$= a^{\frac{3}{2}}b^{\frac{5}{3}}$$

$$\text{e) } (y^{\frac{1}{2}})^{\frac{2}{3}}$$

$$= y^{\frac{2}{6}}$$

$$= y^{\frac{1}{3}}$$

$$\text{f) } (u^{\frac{3}{4}}v^{\frac{1}{2}})^{\frac{2}{9}} = u^{\frac{6}{36}}v^{\frac{2}{18}}$$

$$= u^{\frac{1}{6}}v^{\frac{1}{9}}$$

6) Simplify. Express your answers using only positive exponents.

$$\text{a) } k^{\frac{3}{4}} \div k^{-\frac{1}{4}}$$

$$= k^{\frac{4}{4}}$$

$$= k$$

$$\text{b) } \frac{p^{\frac{2}{3}}}{p^{\frac{5}{6}}} = \frac{p^{\frac{4}{6}}}{p^{\frac{5}{6}}}$$

$$= p^{-\frac{1}{6}}$$

$$= \frac{1}{p^{\frac{1}{6}}}$$

$$\text{c) } (y^{\frac{2}{3}})^{-3}$$

$$= y^{-\frac{6}{3}}$$

$$= y^{-2}$$

$$= \frac{1}{y^2}$$

$$\text{d) } (w^{-\frac{8}{9}})^{\frac{3}{4}}$$

$$= w^{-\frac{24}{36}}$$

$$= w^{-\frac{2}{3}}$$

$$\text{e) } (8x)^{\frac{2}{3}}(27x)^{-\frac{1}{3}}$$

$$= \frac{4x^{\frac{2}{3}}}{(27x)^{\frac{1}{3}}}$$

$$= \frac{4x^{\frac{2}{3}}}{3x^{\frac{1}{3}}}$$

$$= \frac{4x^{\frac{1}{3}}}{3}$$

$$\text{f) } 5(7y^{-\frac{2}{3}})^{-2}$$

$$= 5(7^{-2})(y^{\frac{4}{3}})$$

$$= \frac{5y^{\frac{4}{3}}}{49}$$

7) The surface area, S, of a sphere can be expressed in terms of its volume, V, using the formula

$S(V) = (4\pi)^{\frac{1}{3}}(3V)^{\frac{2}{3}}$. A beach ball has a volume of 24 000 cm³. Find its surface area, to the nearest hundred square centimeters.

$$S(24000) = (4\pi)^{\frac{1}{3}} [3(24000)]^{\frac{2}{3}}$$

$$= (4\pi)^{\frac{1}{3}} (72000)^{\frac{2}{3}}$$

$$= 4023.7 \text{ cm}^2$$

$$\approx 4000 \text{ cm}^2$$

Answers

1. a) 4 b) -10 c) $\frac{1}{2}$ d) $\frac{2}{3}$
2. a) 3 b) $\frac{2}{5}$ c) 2 d) -10
3. a) 4 b) 16 c) -1024 d) $\frac{1}{1000}$
4. a) $\frac{1}{2}$ b) $\frac{1}{125}$ c) 128
d) 4 e) $\frac{27}{1000}$ f) $\frac{9}{4}$
5. a) $x^{\frac{1}{2}}$ b) $m^{\frac{13}{12}}$ c) $w^{\frac{1}{6}}$
d) $a^{\frac{1}{2}}b^{\frac{5}{3}}$ e) $y^{\frac{1}{3}}$ f) $u^{\frac{1}{8}}v^{\frac{1}{9}}$
6. a) k b) $\frac{1}{p^{\frac{3}{2}}}$ c) $\frac{1}{y^{\frac{1}{2}}}$
d) $w^{\frac{2}{3}}$ e) $\frac{4}{3}x^{\frac{1}{3}}$ f) $\frac{5}{49}y^{\frac{4}{3}}$
7. 4000 cm²

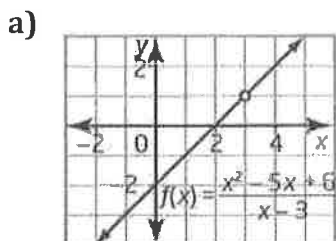
2.1/2.2 Multiplying and Dividing Rational Expressions - Worksheet

MCR3U

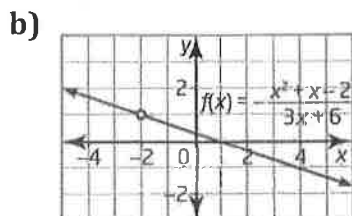
Jensen

SOLUTIONS

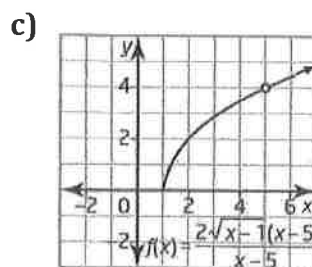
1) State the restrictions for each function.



$$x \neq 3$$

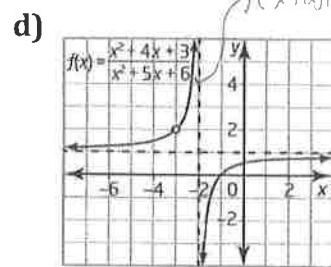


$$x \neq -2$$



$$x \neq 5$$

$$x \geq 1$$



$$x \neq -2, -3$$

2) Simplify each expression and state all restrictions on x .

a) $\frac{x-8}{x^2-13x+40}$

$$= \frac{x-8}{(x-5)(x-8)}$$

$$= \frac{1}{x-5}, x \neq 5, 8$$

b) $\frac{x^2-3x-18}{x^2+x-42}$

$$= \frac{(x-6)(x+3)}{(x+7)(x-6)}$$

$$= \frac{x+3}{x+7}, x \neq -7, 6$$

c) $\frac{x+8}{x^2+6x-16}$

$$= \frac{x+8}{(x+8)(x-2)}$$

$$= \frac{1}{x-2}, x \neq -8, 2$$

3) Simplify and state the restrictions on the variables.

a) $\frac{14y}{11x} \times \frac{121y}{7x}$

$$= \frac{22y^2}{x^2}, x \neq 0$$

b) $\frac{15b^3}{4b} \times \frac{20b}{30b^2}$

$$= \frac{5b^4}{2b^3}$$

$$= \frac{5b}{2}, b \neq 0$$

c) $\frac{5x}{9y} \div \frac{5x}{18y^2}$

$$= \frac{5x}{9y} \times \frac{18y^2}{5x}$$

$$= 2y, x \neq 0, y \neq 0$$

d) $\frac{26ab}{4a} \div \frac{39a^4b^3}{12b^4}$

$$= \frac{26ab}{4a} \times \frac{12b^4}{39a^4b^3}$$

$$= \frac{6ab^5}{3a^5b^3}$$

$$= \frac{2b^2}{a^4}, a \neq 0, b \neq 0$$

4) Simplify and state the restrictions on the variable.

a) $\frac{5x^5}{x+10} \times \frac{x+10}{5}$

$$= 5, x \neq -10$$

b) $\frac{x+5}{x-3} \times \frac{x-3}{x+7}$

$$= \frac{x+5}{x+7}, x \neq 3, -7$$

$$\text{c) } \frac{x+1}{x} \div \frac{x+1}{2x}$$

$$= \frac{\cancel{x+1}}{\cancel{x}} \cdot \frac{2\cancel{x}}{\cancel{x+1}}$$

$$= 2, x \neq 0, -1$$

$$\text{d) } \frac{x+12}{x+10} \div \frac{x+12}{x-5}$$

$$= \frac{\cancel{x+12}}{\cancel{x+10}} \cdot \frac{\cancel{x-5}}{\cancel{x+12}}$$

$$= \frac{x-5}{x+10}, x \neq -10, -12, 5$$

5) Simplify and state the restrictions on the variable.

$$\text{a) } \frac{3x^2}{12x^2+18x} \times \frac{4x+6}{3x+30}$$

$$= \frac{\cancel{3}x^2}{6\cancel{x}(x+3)} \cdot \frac{2\cancel{(x+3)}}{\cancel{3}(x+10)}$$

$$= \frac{\cancel{2}x^2}{3\cancel{6}(x+10)} = \frac{x}{3(x+10)}, x \neq -10, -\frac{3}{2}, 0$$

$$\text{b) } \frac{4x+24}{x^2+8x} \times \frac{12x^2}{3x+18}$$

$$= \frac{4\cancel{(x+6)}}{\cancel{x}(x+8)} \cdot \frac{4\cancel{3}x^2}{\cancel{3}(x+6)}$$

$$= \frac{16x^2}{\cancel{x}(x+8)} = \frac{16x}{x+8}, x \neq -8, -6, 0$$

$$\text{c) } \frac{x^2+10x+21}{x+3} \times \frac{x+2}{x^2+9x+14}$$

$$= \frac{\cancel{(x+7)}\cancel{(x+3)}}{\cancel{x+3}} \cdot \frac{\cancel{x+2}}{\cancel{(x+2)}\cancel{(x+7)}}$$

$$= 1, x \neq -3, -2, -7$$

$$\text{d) } \frac{x^2+2x-15}{x^2-9x+18} \times \frac{x-6}{x+5}$$

$$= \frac{\cancel{(x+5)}\cancel{(x-3)}}{\cancel{(x-6)}\cancel{(x-3)}} \cdot \frac{\cancel{(x-6)}}{\cancel{(x+5)}}$$

$$= 1, x \neq -5, 3, 6$$

6) Simplify and state the restrictions on the variable

$$\text{a) } \frac{x^2+15x}{4x+24} \div \frac{3x}{3x+18}$$

$$= \frac{\cancel{x}(x+15)}{4\cancel{(x+6)}} \times \frac{\cancel{3}(x+6)}{\cancel{3}x}$$

$$= \frac{x+15}{4}, x \neq -6, 0$$

$$\text{b) } \frac{6x}{8x-72} \div \frac{9x}{2x-18}$$

$$= \frac{\cancel{2}6x}{8\cancel{(x-9)}} \cdot \frac{2\cancel{(x-9)}}{\cancel{3}9x}$$

$$= \frac{4}{24}$$

$$= \frac{1}{6}, x \neq 0, 9$$

$$c) \frac{x^2+15x+26}{6x^2} \div \frac{x^2-3x-10}{30x^3}$$

$$= \frac{\cancel{(x+2)}(x+13)}{6x^2} \times \frac{530x^3}{\cancel{(x-5)}(x+2)}$$

$$= \frac{5x(x+13)}{x-5}, x \neq -2, 0, 5$$

$$d) \frac{x^2+11x+24}{x^2+2x-3} \div \frac{x-8}{x-1}$$

$$= \frac{(x+8)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)} \cdot \frac{x-1}{x-8}$$

$$= \frac{x+8}{x-8}, x \neq -3, 1, 8$$

7) Simplify and state the restrictions on the variable

$$a) \frac{a^2-25}{a+2} \cdot \frac{a^2-4}{a^2-7a+10}$$

$$= \frac{(a-5)\cancel{(a+5)}}{a+2} \cdot \frac{\cancel{(a-2)}(a+2)}{\cancel{(a-2)}(a-5)}$$

$$= a+5, a \neq -2, 2, 5$$

$$b) \frac{y^2-4y-21}{3y^2+6y} \cdot \frac{y^2+8y}{y^2+11y+24}$$

$$= \frac{(y-7)\cancel{(y+3)}}{3y(y+2)} \cdot \frac{y\cancel{(y+8)}}{\cancel{(y+8)}(y+3)}$$

$$= \frac{y-7}{3(y+2)}, y \neq -4, -3, -2, 0$$

$$c) \frac{p^2-2p+1}{p+1} \div \frac{p^2-1}{p+1}$$

$$= \frac{(p-1)\cancel{(p-1)}}{p+1} \times \frac{p+1}{\cancel{(p-1)}(p+1)}$$

$$= \frac{p-1}{p+1}, p \neq -1, 1$$

$$d) \frac{x^2+6x-27}{x^2+11x+18} \div \frac{x-3}{x^2+x-2}$$

$$= \frac{(x+9)\cancel{(x-3)}}{\cancel{(x+9)}(x+2)} \div \frac{x-3}{\cancel{(x+2)}(x-1)}$$

$$= \frac{\cancel{(x+9)}(x-3)}{\cancel{(x+9)}(x+2)} \cdot \frac{\cancel{(x+2)}(x-1)}{x-3}$$

$$= x-1, x \neq -9, -2, 1, 3$$

Answers

1) a) $x \neq 3$ b) $x \neq -2$ c) $x \geq 1, x \neq 5$ d) $x \neq -3, x \neq -2$

2) a) $\frac{1}{x-5}, x \neq 5, x \neq 8$ b) $\frac{x+3}{x+7}, x \neq -7, x \neq 6$ c) $\frac{1}{x-2}, x \neq -8, x \neq 2$

3) a) $\frac{22y^2}{x^2}, x \neq 0$ b) $\frac{5b}{2}, b \neq 0$ c) $2y, x \neq 0, y \neq 0$ d) $\frac{2b^2}{a^4}, a \neq 0, b \neq 0$

4) a) $5, x \neq -10$ b) $\frac{x+5}{x+7}, x \neq -7, x \neq 3$ c) $2, x \neq -1, x \neq 0$ d) $\frac{x-5}{x+10}, x \neq -12, x \neq -10, x \neq 5$

5) a) $\frac{x}{3(x+10)}, x \neq -10, -\frac{3}{2}, 0$ b) $\frac{16x}{x+8}, x \neq -8, -6, 0$ c) $1, x \neq -7, -3, -2$ d) $1, x \neq -5, 3, 6$

6) a) $\frac{x+15}{4}, x \neq -6, 0$ b) $\frac{1}{6}, x \neq 0, 9$ c) $\frac{5x(x+13)}{x-5}, x \neq -2, 0, 5$ d) $\frac{x+8}{x-8}, x \neq -3, 1, 8$

7) a) $a + 5, a \neq 2, -2, 5$ b) $\frac{y-7}{3(y+2)}, y \neq -8, -3, -2, 0$ c) $\frac{p-1}{p+1}, p \neq -1, 1$ d) $x - 1, x \neq -9, -2, 1, 3$

2.1/2.2 Adding and Subtracting Rational Expressions - Worksheet

MCR3U

Jensen

SOLUTIONS

1) Simplify and state any restrictions

$$\text{a) } \frac{x+1}{18} + \frac{x-1}{45}$$

$$= \frac{5(x+1) + 2(x-1)}{90}$$

$$= \frac{5x+5+2x-2}{90}$$

$$= \frac{7x+3}{90}$$

$$\text{b) } \frac{2}{3x} - \frac{1}{4x}$$

$$= \frac{8}{12x} - \frac{3}{12x}$$

$$= \frac{5}{12x}, x \neq 0$$

$$\text{c) } \frac{3}{ab} + \frac{5}{4b}$$

$$= \frac{12}{4ab} + \frac{5a}{4ab}$$

$$= \frac{12+5a}{4ab}, a \neq 0, b \neq 0$$

$$\text{d) } \frac{2+a}{a^2b} + \frac{4-a}{3ab^2}$$

$$= \frac{3b(2+a)}{3a^2b^2} + \frac{a(4-a)}{3a^2b^2}$$

$$= \frac{6b+3ab+4a-a^2}{3a^2b^2}, a \neq 0, b \neq 0$$

2) Simplify and state the restrictions.

$$\text{a) } \frac{1}{x-6} - \frac{1}{x+6}$$

$$= \frac{1(x+6)}{(x-6)(x+6)} - \frac{(x-6)}{(x-6)(x+6)}$$

$$= \frac{x+6-x+6}{(x-6)(x+6)}$$

$$= \frac{12}{(x-6)(x+6)}, x \neq -6, 6$$

$$\text{b) } \frac{12}{x+8} + \frac{3}{x-9}$$

$$= \frac{12(x-9)}{(x+8)(x-9)} + \frac{3(x+8)}{(x+8)(x-9)}$$

$$= \frac{12x-108+3x+24}{(x+8)(x-9)}$$

$$= \frac{15x-84}{(x+8)(x-9)}, x \neq -8, 9$$

$$c) \frac{x+10}{x-6} - \frac{x-3}{x+4}$$

$$= \frac{(x+4)(x+10)}{(x+4)(x-6)} - \frac{(x-6)(x-3)}{(x+4)(x-6)}$$

$$= \frac{x^2 + 14x + 40 - (x^2 - 9x + 18)}{(x+4)(x-6)}$$

$$= \frac{23x + 22}{(x+4)(x-6)} \quad , x \neq -4, 6$$

$$d) \frac{x+5}{x+1} + \frac{x+2}{x-2}$$

$$\frac{(x+5)(x-2)}{(x+1)(x-2)} + \frac{(x+1)(x+2)}{(x+1)(x-2)}$$

$$= \frac{x^2 + 3x - 10}{(x+1)(x-2)} + \frac{x^2 + 3x + 2}{(x+1)(x-2)}$$

$$= \frac{2x^2 + 6x - 8}{(x+1)(x-2)}$$

$$\rightarrow \frac{2(x^2 + 3x - 4)}{(x+1)(x-2)}$$

$$= \frac{2(x+4)(x-1)}{(x+1)(x-2)} \quad , x \neq -1, 2$$

3) Simplify and state the restrictions.

$$a) \frac{x}{x^2 - 9x + 8} + \frac{2}{x-8}$$

$$= \frac{x}{(x-8)(x-1)} + \frac{2}{x-8}$$

$$= \frac{x}{(x-8)(x-1)} + \frac{2(x-1)}{(x-8)(x-1)}$$

$$= \frac{x + 2x - 2}{(x-8)(x-1)}$$

$$= \frac{3x - 2}{(x-8)(x-1)} \quad , x \neq 8, 1$$

$$b) \frac{x+3}{x+5} + \frac{x+2}{x^2 + 3x - 10}$$

$$= \frac{x+3}{x+5} + \frac{x+2}{(x+5)(x-2)}$$

$$= \frac{(x+3)(x-2)}{(x+5)(x-2)} + \frac{x+2}{(x+5)(x-2)}$$

$$= \frac{x^2 + x - 6 + x + 2}{(x+5)(x-2)}$$

$$= \frac{x^2 + 2x - 4}{(x+5)(x-2)} \quad , x \neq -5, 2$$

$$c) \frac{x}{x^2 + 3x + 2} - \frac{3x-2}{x^2 + 8x + 7}$$

$$= \frac{x}{(x+2)(x+1)} - \frac{3x-2}{(x+7)(x+1)}$$

$$= \frac{x(x+7)}{(x+2)(x+1)(x+7)} - \frac{(3x-2)(x+2)}{(x+2)(x+1)(x+7)}$$

$$= \frac{x^2 + 7x - (3x^2 + 4x - 4)}{(x+2)(x+1)(x+7)}$$

$$= \frac{-2x^2 + 3x + 4}{(x+2)(x+1)(x+7)} \quad , x \neq -2, -1, -7$$

$$d) \frac{x+4}{x^2 - 121} - \frac{2x-1}{x^2 + 8x - 33}$$

$$= \frac{x+4}{(x-11)(x+11)} - \frac{2x-1}{(x+11)(x-3)}$$

$$= \frac{(x+4)(x-3)}{(x-11)(x+11)(x-3)} - \frac{(2x-1)(x-11)}{(x-11)(x+11)(x-3)}$$

$$= \frac{x^2 + x - 12 - (2x^2 - 23x + 11)}{(x-11)(x+11)(x-3)}$$

$$= \frac{-1x^2 + 24x - 23}{(x-11)(x+11)(x-3)} \quad \rightarrow \text{see bottom of next page...}$$

4) Binomial expressions can differ by a factor of -1. Factor -1 from one of the denominators to identify the common denominator. Then, simplify each expression and state the restrictions.

$$\begin{aligned} & \frac{1}{x-2} - \frac{1}{2-x} \\ &= \frac{-1(1)}{-1(x-2)} - \frac{1}{2-x} \\ &= \frac{-1}{-x+2} - \frac{1}{2-x} \\ &= \frac{-1}{2-x} - \frac{1}{2-x} \\ &= \frac{-2}{2-x} \quad , x \neq 2 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{2b+3}{4b-1} + \frac{b+6}{1-4b} \\ &= \frac{2b+3}{4b-1} + \frac{-1(b+6)}{-1(1-4b)} \\ &= \frac{2b+3}{4b-1} + \frac{-b-6}{-1+4b} \\ &= \frac{2b+3}{4b-1} + \frac{-b-6}{4b-1} \\ &= \frac{2b+3-b-6}{4b-1} \\ &= \frac{b-3}{4b-1} \quad , b \neq \frac{1}{4} \end{aligned}$$

$$\begin{aligned} 3 \text{ d) } &= \frac{-1(x^2-24x+23)}{(x-11)(x+11)(x-3)} \\ &= \frac{-1(x-23)(x-1)}{(x-11)(x+11)(x-3)} \quad , x \neq -11, 3, 11 \end{aligned}$$

Answers

- 1) a) $\frac{7x+3}{90}$, no restrictions b) $\frac{5}{12x}$, $x \neq 0$ c) $\frac{12+5a}{4ab}$, $a \neq 0, b \neq 0$ d) $\frac{6b+3ab+4a-a^2}{3a^2b^2}$, $a \neq 0, b \neq 0$
- 2) a) $\frac{12}{(x-6)(x+6)}$, $x \neq -6, 6$ b) $\frac{15x-84}{(x+8)(x-9)}$, $x \neq -8, 9$ c) $\frac{23x+22}{(x-6)(x+4)}$, $x \neq -4, x \neq 6$ d) $\frac{2(x+4)(x-1)}{(x+1)(x-2)}$, $x \neq -1, 2$
- 3) a) $\frac{3x-2}{(x-1)(x-8)}$, $x \neq 1, 8$ b) $\frac{x^2+2x-4}{(x+5)(x-2)}$, $x \neq -5, 2$ c) $\frac{-2x^2+3x+4}{(x+1)(x+2)(x+7)}$, $x \neq -7, -2, -1$ d) $\frac{-(x-23)(x-1)}{(x+11)(x-11)(x-3)}$, $x \neq -11, 3, 11$
- 4) a) $\frac{2}{x-2}$, $x \neq 2$ b) $\frac{b-3}{4b-1}$, $b \neq \frac{1}{4}$

Chapter 2a - Rational Expressions - REVIEW

MCR3U

Jensen

SOLUTIONS

Section 1: Negative and Rational Exponents

1) Evaluate. Express as a fraction in lowest terms.

a) 10^{-1}

$$= \frac{1}{10}$$

b) 4^{-2}

$$= \frac{1}{4^2}$$

$$= \frac{1}{16}$$

c) $3^{-2} + 9^{-1}$

$$= \frac{1}{9} + \frac{1}{9}$$

$$= \frac{2}{9}$$

d) $5^{-3} + 5^0$

$$= \frac{1}{125} + 1$$

$$= \frac{126}{125}$$

e) $\left(\frac{1}{5}\right)^{-1}$

$$= \left(\frac{5}{1}\right)^1$$

$$= 5$$

f) $\left(\frac{3}{4}\right)^{-3}$

$$= \left(\frac{4}{3}\right)^3$$

$$= \frac{64}{27}$$

2) Simplify. Express your answers using only positive exponents.

a) $(x^{-2})(x^{-1})(x^0)$

$$= x^{-3}$$

$$= \frac{1}{x^3}$$

b) $(3km^2)(2k^{-2}m^{-2})$

$$= 6k^{-1}m^0$$

$$= \frac{6}{k}$$

c) $w^{-3} \div w^{-2}$

$$= w^{-1}$$

$$= \frac{1}{w}$$

d) $\frac{u^{-2}v^3}{u^{-3}v^{-2}}$

$$= uv^5$$

e) $(z^{-3})^{-2}$

$$= z^{-6}$$

$$= \frac{1}{z^6}$$

f) $(2ab^{-1})^{-2}$

$$= \frac{1}{(2ab^{-1})^2}$$

$$= \frac{1}{4a^2b^{-2}}$$

$$= \frac{b^2}{4a^2}$$

3) Simplify. Express your answers using only positive exponents.

$$\begin{aligned} \text{a) } (4a^{-2})(-2a^{-3}) \\ &= -8a^{-5} \\ &= \frac{-8}{a^5} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{(2x^2y)^{-2}(3xy)^{-1}}{(6x^2y^2)^{-2}} &= \frac{36x^4y^4}{12x^5y^3} \\ &= \frac{(6x^2y^2)^2}{(2x^2y)^2(3xy)^1} = 3x^{-1}y \\ &= \frac{36x^4y^4}{4x^4y^2(3xy)} = \frac{3y}{x} \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{1}{4x^2}\right)^{-2} \\ &= \left(\frac{4x^2}{1}\right)^2 \\ &= 16x^4 \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{6a^3}{4b^4}\right)^{-2} \\ &= \left(\frac{4b^4}{6a^3}\right)^2 \\ &= \frac{16b^8}{36a^6} \\ &= \frac{4b^8}{9a^6} \end{aligned}$$

4) Evaluate.

$$\begin{aligned} \text{a) } \sqrt[3]{64} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt[4]{625} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt[5]{-3125} \\ &= -5 \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{1}{64}\right)^{\frac{1}{6}} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{e) } 27^{\frac{2}{3}} \\ &= \left(27^{\frac{1}{3}}\right)^2 \\ &= (3)^2 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{f) } (-1000)^{\frac{4}{3}} \\ &= \left[(-1000)^{\frac{1}{3}}\right]^4 \\ &= (-10)^4 \\ &= 10000 \end{aligned}$$

$$\begin{aligned} \text{g) } -4^{-3} \\ &= \frac{-1}{4^3} \\ &= \frac{-1}{64} \end{aligned}$$

$$\begin{aligned} \text{h) } \left(\frac{3}{4}\right)^{-2} \\ &= \left(\frac{4}{3}\right)^2 \\ &= \frac{16}{9} \end{aligned}$$

$$\begin{aligned} \text{i) } \left(-\frac{27}{125}\right)^{-\frac{2}{3}} \\ &= \left[\left(-\frac{125}{27}\right)^{\frac{1}{3}}\right]^2 \\ &= \left(\frac{-5}{3}\right)^2 \\ &= \frac{25}{9} \end{aligned}$$

5) Simplify. Express answers using only positive exponents.

$$\begin{aligned} \text{a) } n^{\frac{1}{2}} \times n^{\frac{1}{3}} \times n^{\frac{1}{4}} \\ = \frac{1}{n^{\frac{1}{9}}} \end{aligned}$$

$$\begin{aligned} \text{b) } (27y^3)^{\frac{1}{3}} \times \left(\frac{1}{16y^4}\right)^{-\frac{3}{4}} \\ = 3y \times \left(\frac{16y^4}{1}\right)^{\frac{3}{4}} \\ = 3y(8y^3) \\ = 24y^4 \end{aligned}$$

$$\begin{aligned} \text{c) } (27x^6)^{\frac{2}{3}} \div (9x^4)^{\frac{1}{2}} \\ = 9x^4 \div 3x^2 \\ = 3x^2 \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{x^{\frac{2}{3}}}{x^{\frac{4}{5}}} \\ = \frac{x^{\frac{4}{15}}}{x^{\frac{8}{15}}} \\ = \frac{x^{\frac{12}{15}}}{x^{\frac{10}{15}}} \\ = x^{\frac{2}{15}} \end{aligned}$$

Section 2: Rational Expressions

6) Simplify each expression and state all restrictions on x .

$$\text{a) } \frac{x+7}{x^2+10x+21}$$

$$= \frac{x+7}{(x+7)(x+3)}$$

$$= \frac{1}{x+3}, \quad x \neq -7, -3$$

$$\text{b) } \frac{x^2-64}{x-8}$$

$$= \frac{(x-8)(x+8)}{x-8}$$

$$= x+8, \quad x \neq 8$$

$$\text{c) } \frac{x^2-9}{x^2-8x+15}$$

$$= \frac{(x-3)(x+3)}{(x-3)(x-5)}$$

$$= \frac{x+3}{x-5}, \quad x \neq 3, 5$$

7) Simplify each expression and state the restrictions.

$$\text{a) } \frac{3x^2}{5xy} \times \frac{20xy^3}{12xy}$$

$$= \frac{60x^3y^3}{60x^2y^2}$$

$$= xy; \quad x \neq 0, \quad y \neq 0$$

$$\text{b) } \frac{150a^3b^4}{20a^2b} \div \frac{6b}{8ab^2}$$

$$= \frac{150a^3b^4}{20a^2b} \times \frac{4ab^2}{3b}$$

$$= \frac{60a^4b^6}{6a^2b^2}$$

$$= 10a^2b^4; \quad a \neq 0, \quad b \neq 0$$

$$c) \frac{1}{3x} + \frac{5}{2x^2}$$

$$= \frac{2x(1)}{2x(3x)} + \frac{3(5)}{3(2x^2)}$$

$$= \frac{2x}{6x^2} + \frac{15}{6x^2}$$

$$= \frac{2x+15}{6x^2} ; x \neq 0$$

8) Simplify each expression and state restrictions.

$$a) \frac{x^2+7x}{3x+21} \times \frac{x^2+3x+2}{x+2}$$

$$= \frac{x(x+7)}{3(x+7)} \cdot \frac{(x+2)(x+1)}{(x+2)}$$

$$= \frac{x(x+1)}{3} ; x \neq -7, -2$$

$$d) \frac{4}{x-6} - \frac{3}{x-4}$$

$$= \frac{4(x-4)}{(x-6)(x-4)} - \frac{3(x-6)}{(x-6)(x-4)}$$

$$= \frac{4x-16-3x+18}{(x-6)(x-4)}$$

$$= \frac{x+2}{(x-6)(x-4)} ; x \neq 6, 4$$

$$b) \frac{x^2+4x-60}{3x+30} \div \frac{x^2-8x+12}{6x-12}$$

$$= \frac{(x+10)(x-6)}{3(x+10)} \times \frac{6(x-2)}{(x-2)(x-2)}$$

$$= \frac{6}{3}$$

$$= 2 ; x \neq -10, 2, 6$$

$$c) \frac{3}{x^2+7x+10} - \frac{5x}{x^2-4}$$

$$= \frac{3}{(x+2)(x+5)} - \frac{5x}{(x-2)(x+2)}$$

$$= \frac{3(x-2)}{(x+2)(x+5)(x-2)} - \frac{5x(x+5)}{(x+2)(x+5)(x-2)}$$

$$= \frac{3x-6-5x^2-25x}{(x+2)(x+5)(x-2)}$$

$$= \frac{-5x^2-22x-6}{(x+2)(x+5)(x-2)} ; x \neq -5, -2, 2$$

$$d) \frac{-10x}{x^2+18x+32} + \frac{12x}{x^2+6x-160}$$

$$= \frac{-10x}{(x+16)(x+2)} + \frac{12x}{(x+16)(x-10)}$$

$$= \frac{-10x(x+10)}{(x+16)(x+2)(x-10)} + \frac{12x(x+2)}{(x+16)(x+2)(x-10)}$$

$$= \frac{-10x^2+100x+12x^2+24x}{(x+16)(x+2)(x-10)}$$

$$= \frac{2x^2+124x}{(x+16)(x+2)(x-10)}$$

$$= \frac{2x(x+62)}{(x+16)(x+2)(x-10)} ; x \neq -16, -2, 10$$

9) Simplify each expression and state any restrictions

a) $\frac{x-8}{x+7} \times \frac{x+15}{x^2+12x-45}$

$$= \frac{x-8}{x+7} \cdot \frac{x+15}{(x+15)(x-3)}$$

$$= \frac{x-8}{(x+7)(x-3)} ; x \neq -7, 3$$

b) $\frac{x^2+12x+20}{x+5} \div \frac{x^2+7x-30}{x+10}$

$$= \frac{(x+10)(x+2)}{x+5} \cdot \frac{x+10}{(x+10)(x-3)}$$

$$= \frac{(x+10)(x+2)}{(x+5)(x-3)} ; x \neq -10, -5, 3$$

c) $\frac{x+3}{x-7} - \frac{x+9}{x-2}$

$$= \frac{(x+3)(x-2)}{(x-7)(x-2)} - \frac{(x+9)(x+7)}{(x-7)(x-2)}$$

$$= \frac{x^2+x-6 - (x^2+2x-63)}{(x-7)(x-2)}$$

$$= \frac{-1x+57}{(x-7)(x-2)} ; x \neq 2, 7$$

e) $\frac{5x+1}{2x-1} - \frac{3x-3}{1-2x}$

$$= \frac{-1(5x+1)}{-1(2x-1)} - \frac{3x-3}{1-2x}$$

$$= \frac{-5x-1}{-2x+1} - \frac{3x-3}{1-2x}$$

$$= \frac{-5x-1}{1-2x} - \frac{3x-3}{1-2x}$$

$$= \frac{-5x-1-(3x-3)}{1-2x}$$

$$= \frac{-8x+2}{1-2x}$$

$$\frac{-2(4x-1)}{1-2x} ; x \neq \frac{1}{2}$$

d) $\frac{x+8}{x+3} + \frac{x-6}{x^2+9x+18}$

$$= \frac{x+8}{x+3} + \frac{x-6}{(x+3)(x+6)}$$

$$= \frac{(x+8)(x+6)}{(x+3)(x+6)} + \frac{x-6}{(x+3)(x+6)}$$

$$= \frac{x^2+14x+48+x-6}{(x+3)(x+6)}$$

$$= \frac{x^2+15x+42}{(x+3)(x+6)} ; x \neq -3, -6$$

Answers

1) a) $\frac{1}{10}$ b) $\frac{1}{16}$ c) $\frac{2}{9}$ d) $\frac{126}{125}$ e) 5 f) $\frac{64}{27}$

2) a) $\frac{1}{x^3}$ b) $\frac{6}{k}$ c) $\frac{1}{w}$ d) uv^5 e) z^6 f) $\frac{b^2}{4a^2}$

3) a) $-\frac{8}{a^5}$ b) $\frac{3y}{x}$ c) $16x^4$ d) $\frac{4b^8}{9a^6}$

4) a) 4 b) 5 c) -5 d) $\frac{1}{2}$ e) 9 f) 10 000 g) $-\frac{1}{64}$ h) $\frac{16}{9}$ i) $\frac{25}{9}$

5) a) $n^{\frac{13}{12}}$ b) $24y^4$ c) $3x^2$ d) $x^{\frac{2}{15}}$

6) a) $\frac{1}{x+3}$, $x \neq -7, x \neq -3$ b) $x+8$, $x \neq 8$ c) $\frac{x+3}{x-5}$, $x \neq 3, 5$

7) a) xy , $x \neq 0, y \neq 0$ b) $10a^2b^4$, $a \neq 0, b \neq 0$ c) $\frac{2x+15}{6x^2}$, $x \neq 0$ d) $\frac{x+2}{(x-4)(x-6)}$, $x \neq 4, x \neq 6$

8) a) $\frac{x(x+1)}{3}$, $x \neq -7, -2$ b) 2 , $x \neq -10, 2, 6$ c) $\frac{-5x^2-22x-6}{(x+5)(x+2)(x-2)}$, $x \neq -5, -2, 2$

d) $\frac{2x(x+62)}{(x+16)(x+2)(x-10)}$, $x \neq -16, -2, 10$

9) a) $\frac{x-8}{(x+7)(x-3)}$, $x \neq -15, -7, 3$ b) $\frac{(x+10)(x+2)}{(x+5)(x-3)}$, $x \neq -10, -5, 3$ c) $\frac{-x+57}{(x-7)(x-2)}$, $x \neq 2, 7$

d) $\frac{x^2+15x+42}{(x+6)(x+3)}$, $x \neq -6, -3$ e) $\frac{8x-2}{2x-1}$, $x \neq \frac{1}{2}$