Chapter 2b – Transformations of Functions – REVIEW SOLUTIONS
MCR3U
Jensen

1) List the transformations of, in words, of $f(x)$ for each of the following functions.

a) $g(x) = 2f(x)$

- vertical stretch bafo 2

b) $h(x) = f(x-3)$

- phase shift right 3 units

c) $j(x) = f\left(\frac{1}{3}x\right)$

- horizontal stretch bafo 3

d) $k(x) = f(-x)$

- horizontal reflection

e) $m(x) = f(x) - 3$

- shift down 3 units.

2) List the transformations, in words, of $f(x)$ for each of the following functions in the order you would do them in.

a) $g(x) = -f(x+2)$

- vertical reflection
- phase shift left 2 units

b) $h(x) = f(3x) + 2$

- horizontal compression bafo $\frac{1}{3}$
- shift up 2 units

c) $j(x) = 3f(-x)$

- vertical stretch bafo 3
- horizontal reflection

3) List the transformations, in words, of $f(x) = x^2$ to $g(x) = 3(x-2)^2 - 11$ in the order you would do them.

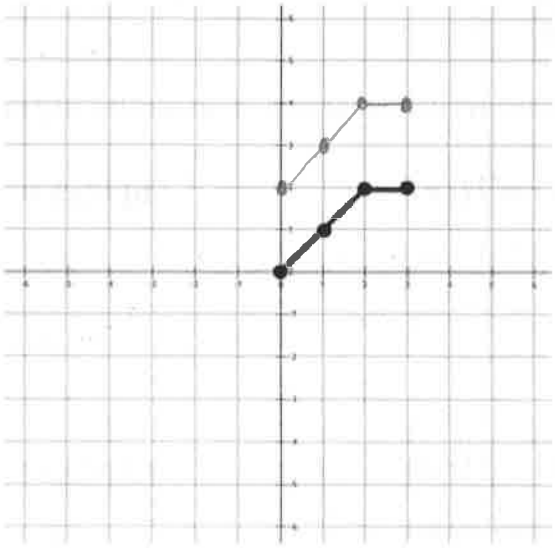
- vertical stretch bafo 3
- phase shift right 2 units
- shift down 11 units

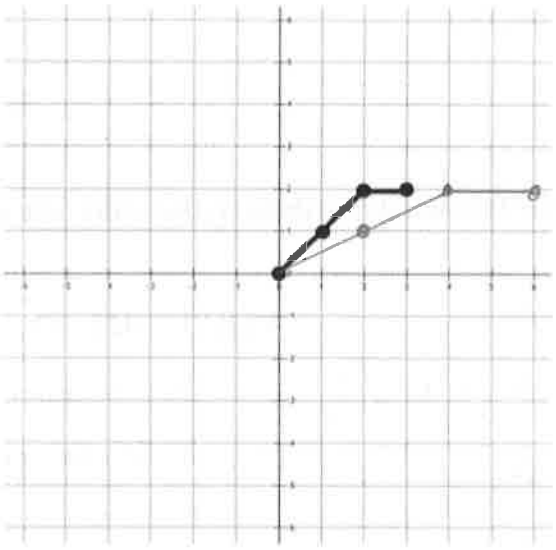
4) List the transformations, in words of $f(x) = \sqrt{x}$ to $g(x) = 2\sqrt{(x-3)} - 9$ in the order you would do them.

- vertical stretch BAFO 2
- phase shift right 3 units
- shift down 9 units

5) Perform the following transformations on the graphs below.

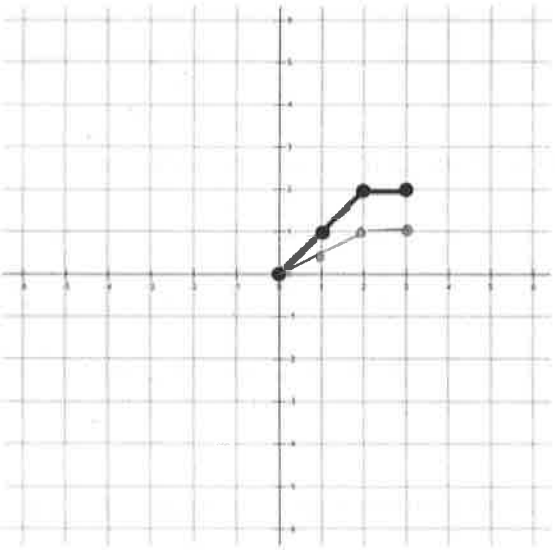
a) translate up 2 units





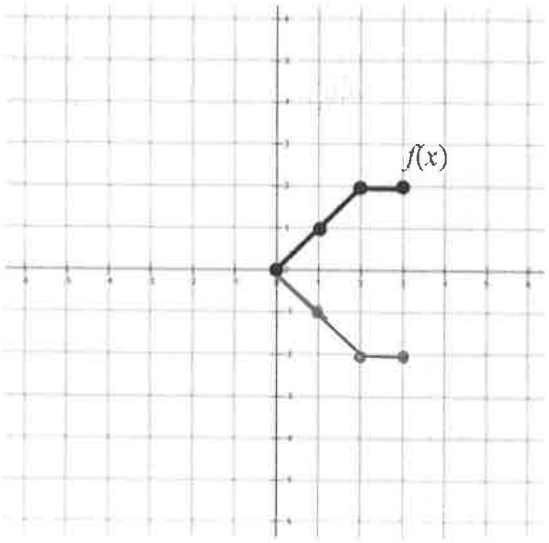
b) horizontal stretch by a factor of 2

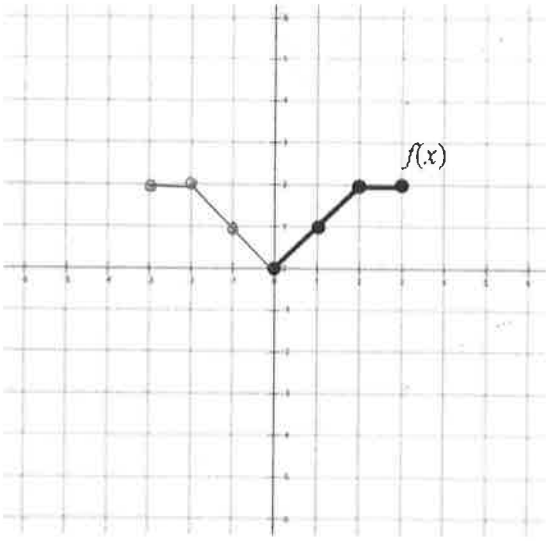
c) vertical ~~stretch~~ compression by a factor of $\frac{1}{2}$



6) For the graph of $f(x)$ given, sketch the graph of $g(x)$ after the given transformation. List the transformations in words as well.

a) $g(x) = -f(x)$





b) $g(x) = f(-x)$

7) For each function $g(x)$:

i) describe the transformations from the parent function $f(x)$
ii) create a table of values of image points for the transformed function
iii) graph the parent function and the transformed function and write its equation

a) $f(x) = x^2$. Graph $g(x) = 2f(x-2)$.

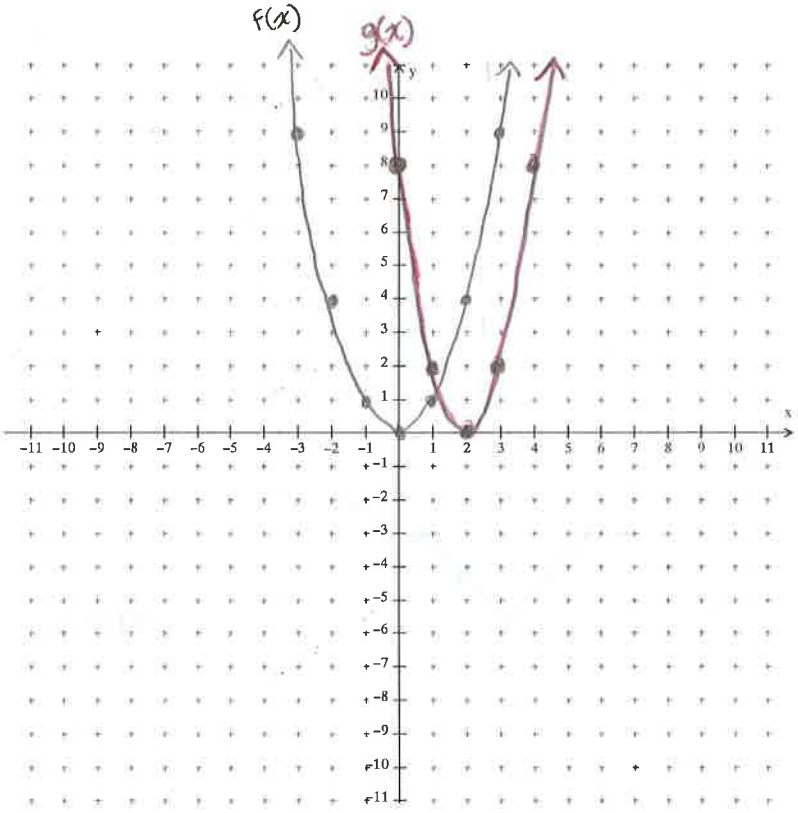
i) vertical stretch BAFO 2 (2y)
shift right 2 units (x+2)

ii)

f(x)	g(x)	
	x+2	2y
(-3,9)	-1	18
(-2,4)	0	8
(-1,1)	1	2
(0,0)	2	0
(1,1)	3	2
(2,4)	4	8
(3,9)	5	18

$f(x) = x^2$

$g(x) = 2(x-2)^2$



b) $f(x) = \sqrt{x}$. Graph $g(x) = -f(2x)$.

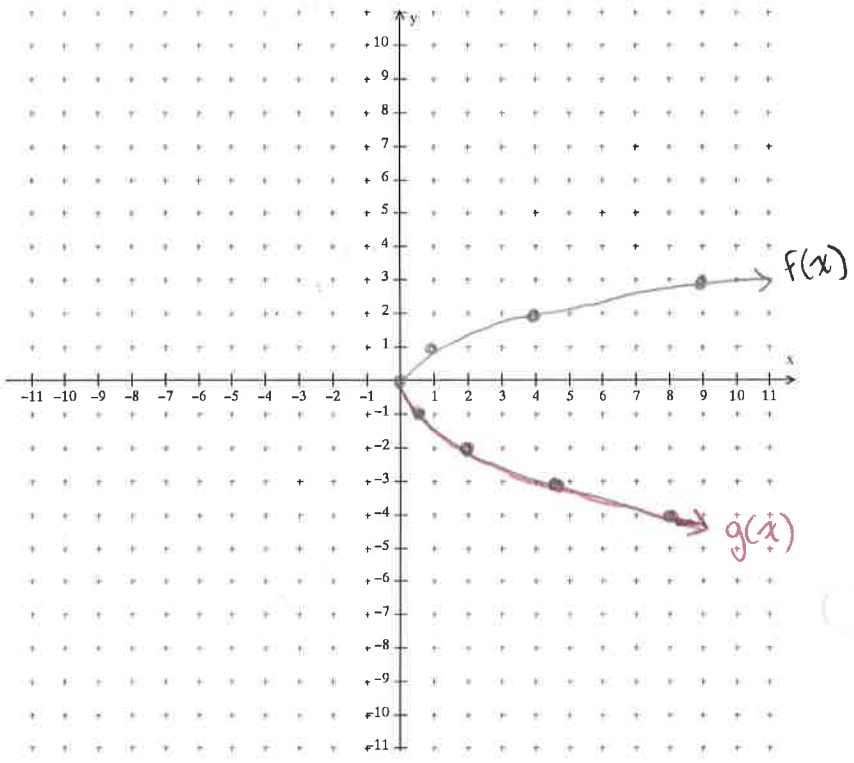
i) vertical reflection (-y)
horizontal compression BAFO $\frac{1}{2}$ $(\frac{x}{2})$

ii)

f(x)	g(x)	
	x/2	-y
(0,0)	0	0
(1,1)	0.5	-1
(4,2)	2	-2
(9,3)	4.5	-3

$f(x) = \sqrt{x}$

$g(x) = -\sqrt{2x}$



c) $f(x) = x^2$. Graph $g(x) = 4f(x-3)$

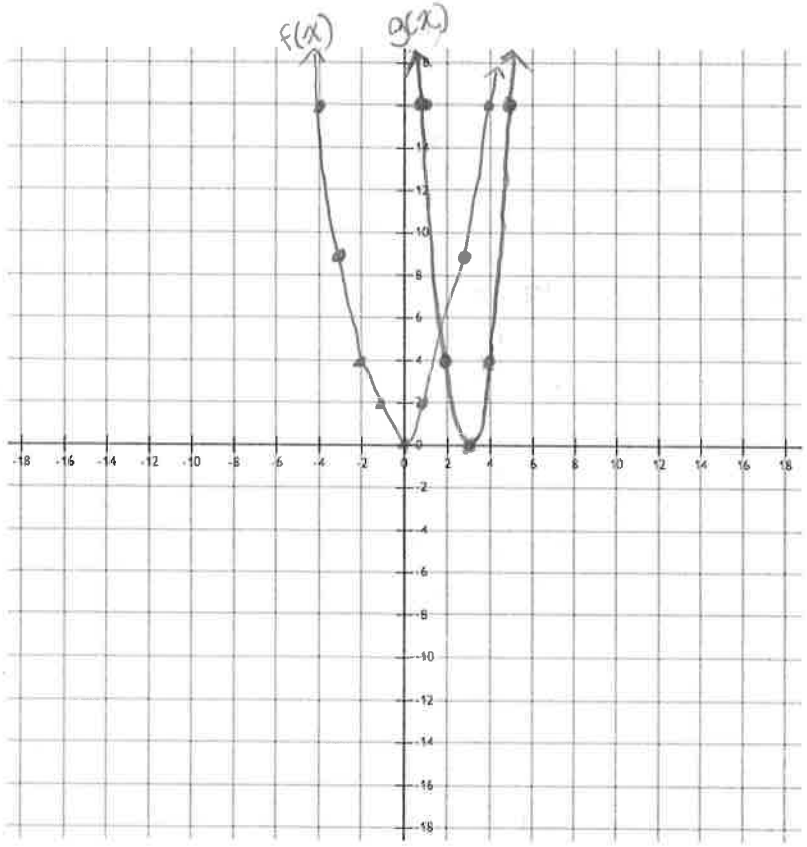
i) vertical stretch BAFO 4 (4y)
shift right 3 units (x-3)

ii)

f(x)	g(x)	
	x+3	4y
(-3, 9)	0	36
(-2, 4)	1	16
(-1, 1)	2	4
(0, 0)	3	0
(1, 1)	4	4
(2, 4)	5	16
(3, 9)	6	36

$f(x) = x^2$

$g(x) = 4(x-3)^2$



d) $f(x) = \sqrt{x}$. Graph $g(x) = 3f(-x) - 2$.

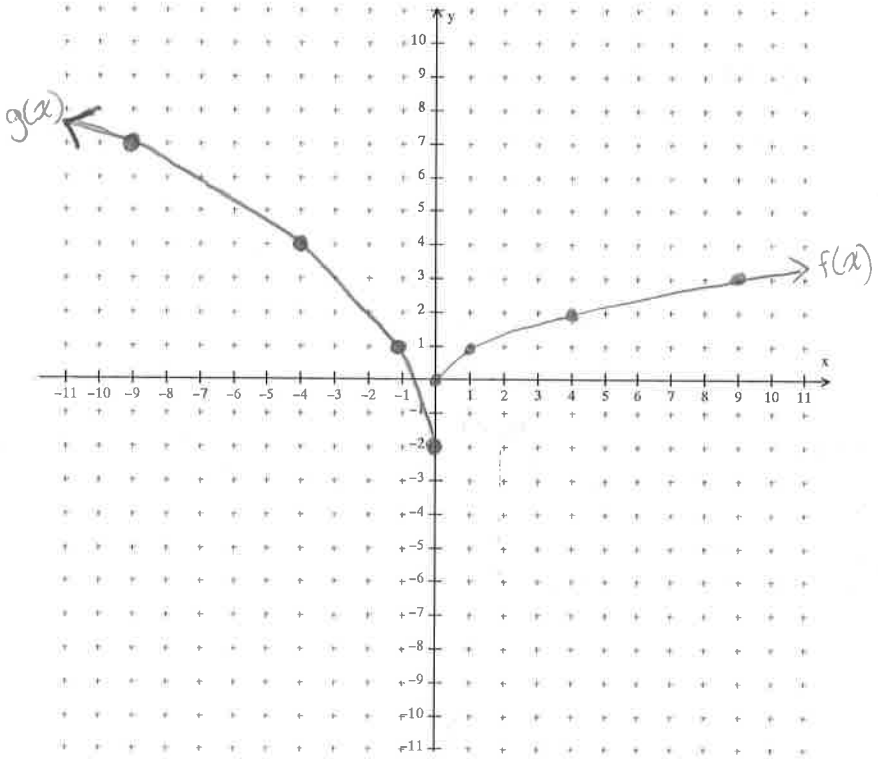
i) vertical stretch BAFO 3 (3y)
horizontal reflection (-x)
shift down 2 units (y-2)

ii)

f(x)	g(x)	
	-x	3y-2
(0, 0)	0	-2
(1, 1)	-1	1
(4, 2)	-4	4
(9, 3)	-9	7

$f(x) = \sqrt{x}$

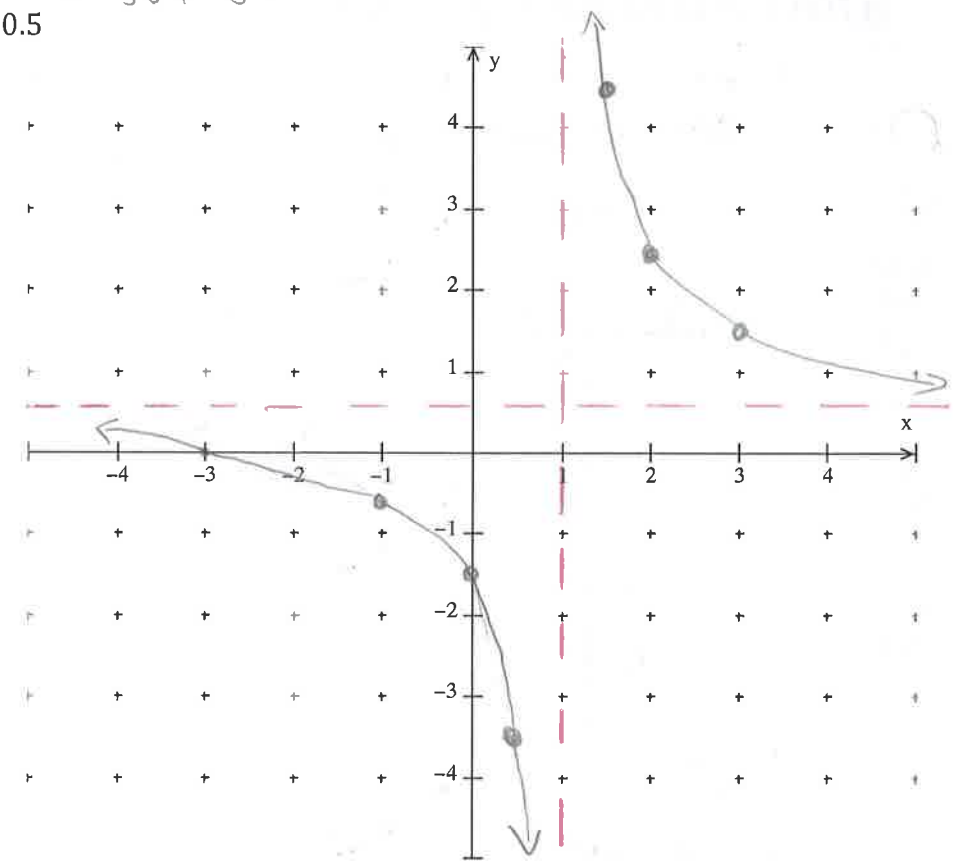
$g(x) = 3\sqrt{-x} - 2$



only graph $g(x)$

e) $f(x) = \frac{1}{x}$. Graph $g(x) = 2f(x-1) + 0.5$

i) vertical stretch factor 2 ($2y$)
shift 1 unit right ($x+1$)
shift up 0.5 units ($y+0.5$)



$f(x)$	$x+1$	$2y+0.5$	$g(x)$
$(-2, -0.5)$	-1	-0.5	$(-1, -0.5)$
$(-1, -1)$	0	-1.5	$(0, -1.5)$
$(-0.5, -2)$	0.5	-3.5	$(0.5, -3.5)$
$(0.5, 2)$	1.5	4.5	$(1.5, 4.5)$
$(1, 1)$	2	2.5	$(2, 2.5)$
$(2, 0.5)$	3	1.5	$(3, 1.5)$

$g(x) = \frac{2}{x-1} + 0.5$

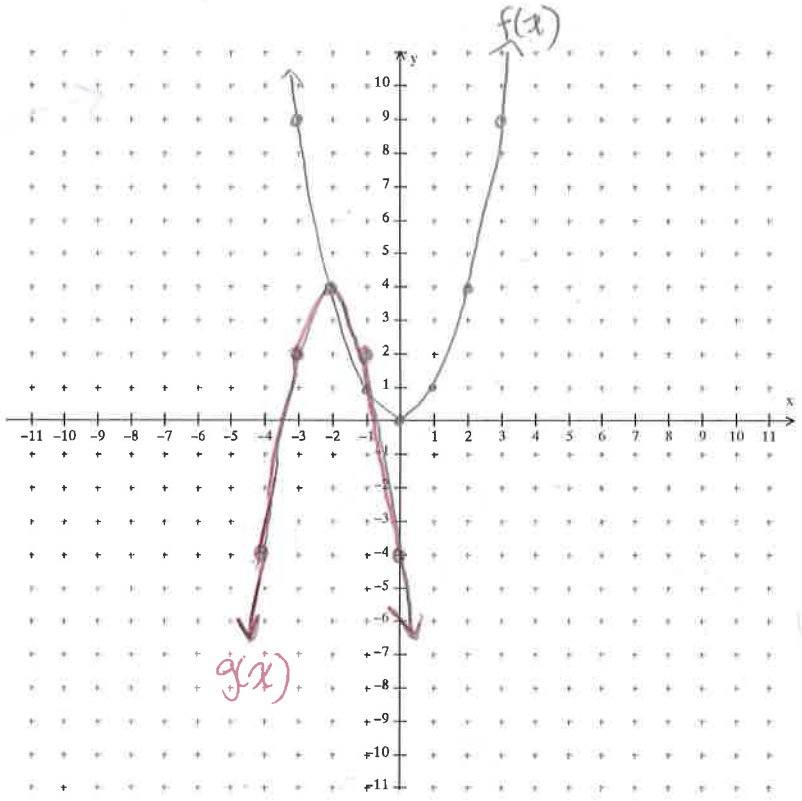
8) For each function $g(x)$:

i) determine the parent function and describe the transformations from the parent function $f(x)$
ii) create a table of values of image points for the transformed function
iii) graph the parent function and the transformed function

a) $g(x) = -2(x+2)^2 + 4$

$f(x) = x^2$

Transformations: vertical stretch BAFO 2 ($2y$)
vertical reflection ($-y$)
left 2 units ($x-2$)
up 4 units ($y+4$)



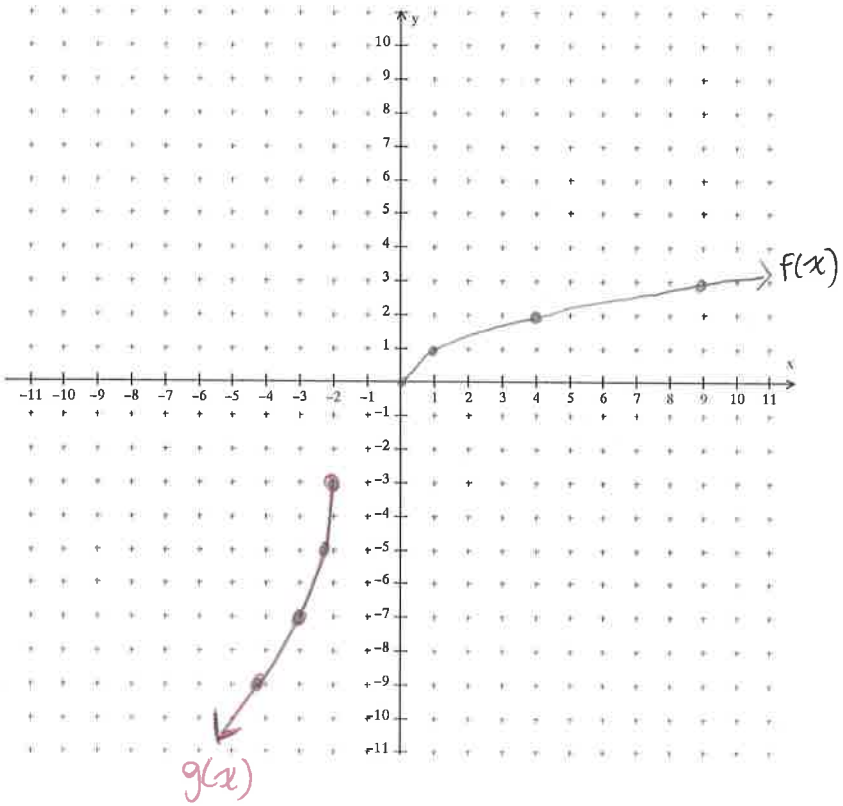
ii)

$f(x)$	$x-2$	$-2y+4$	$g(x)$
$(-3, 9)$	-5	-14	$(-5, -14)$
$(-2, 4)$	-4	-4	$(-4, -4)$
$(-1, 1)$	-3	2	$(-3, 2)$
$(0, 0)$	-2	4	$(-2, 4)$
$(1, 1)$	-1	2	$(-1, 2)$
$(2, 4)$	0	-4	$(0, -4)$
$(3, 9)$	1	-14	$(1, -14)$

$f(x) = \sqrt{x}$

b) $g(x) = -2\sqrt{-4(x+2)} - 3$

i) vertical stretch bafo 2 (2y)
Vertical reflection (-y)
horizontal compression bafo $\frac{1}{4}$ $(\frac{x}{4})$
horizontal reflection (-x)
shift left 2 (x-2)
down 3 (y-3)



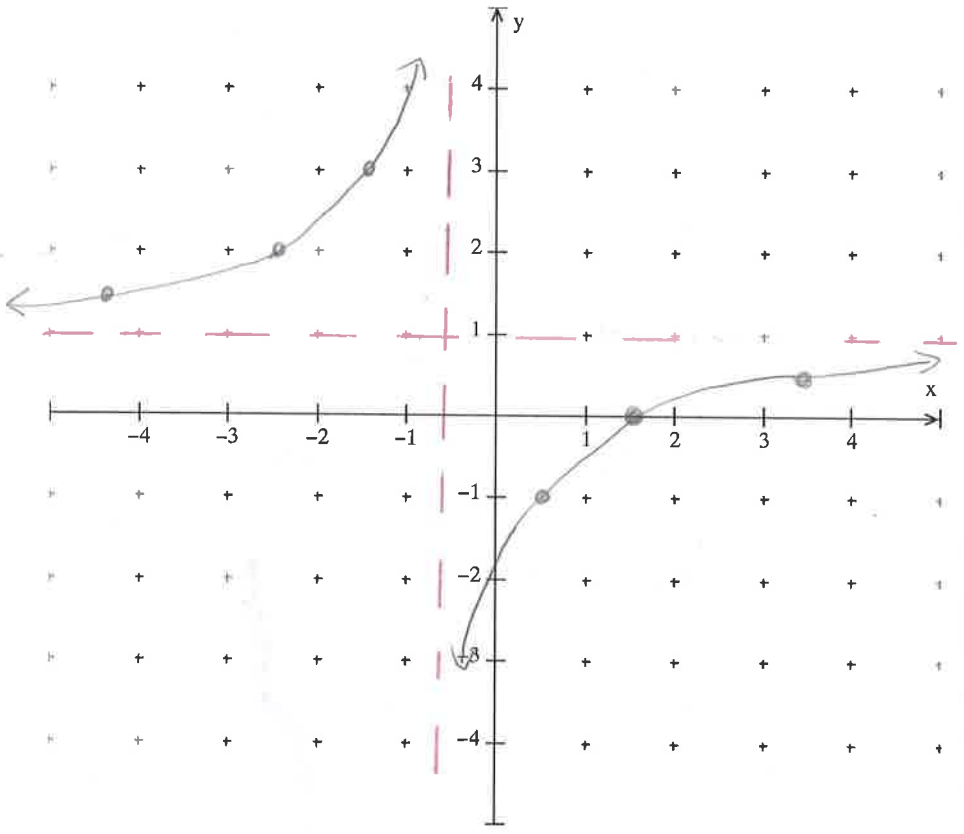
f(x)
(0,0)
(1,1)
(4,2)
(9,3)

g(x)	
$\frac{x}{-4} - 2$	-2y-3
-2	-3
-2.25	-5
-3	-7
-4.25	-9

$f(x) = \frac{1}{x}$

c) $g(x) = \frac{-1}{\frac{1}{2}(x+0.5)} + 1$ only graph g(x)

i) vertical reflection (-y)
horizontal stretch bafo 2 (2x)
left 0.5 units (x-0.5)
up 1 unit (y+1)



f(x)
(-2,-0.5)
(-1,-1)
(-0.5,-2)
(0.5,2)
(1,1)
(2,0.5)

g(x)	
2x-0.5	-y+1
-4.5	1.5
-2.5	2
-1.5	3
0.5	-1
1.5	0
3.5	0.5

9) Find the inverse , $f^{-1}(x)$, algebraically if $f(x) = -2\sqrt{x+1} - 5$

$x = -2\sqrt{y+1} - 5$

$\frac{x+5}{-2} = \sqrt{y+1}$

$\left(\frac{x+5}{-2}\right)^2 = y+1$

$\left(\frac{x+5}{-2}\right)^2 - 1 = y$

$f^{-1}(x) = \left(\frac{x+5}{-2}\right)^2 - 1$

10) Find the inverse , $f^{-1}(x)$, algebraically if $f(x) = \frac{1}{3}(x-4)^2 + 2$

$x = \frac{1}{3}(y-4)^2 + 2$

$x - 2 = \frac{1}{3}(y-4)^2$

$3x - 6 = (y-4)^2$

$\pm\sqrt{3x-6} = y - 4$

$\pm\sqrt{3x-6} + 4 = y$

$f^{-1}(x) = \pm\sqrt{3x-6} + 4$

OR

$x = \frac{1}{3}(y-4)^2 + 2$

$3(x-2) = (y-4)^2$

$\pm\sqrt{3(x-2)} = y - 4$

$\pm\sqrt{3(x-2)} + 4 = y$

$f^{-1}(x) = \pm\sqrt{3(x-2)} + 4$

Answers

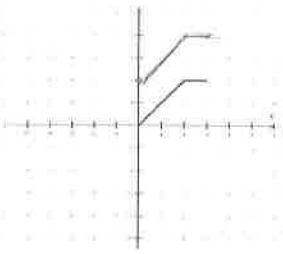
1) a) vertical stretch BAFO 2 **b)** phase shift right 3 units **c)** horizontal stretch BAFO 3
d) horizontal reflection in the y-axis **e)** shift down 3 units

2) a) vertical reflection in the x-axis and then shift left 3 units.
b) horizontal compression BAFO $\frac{1}{3}$ and then shift up 2 units.
c) vertical stretch BAFO 3 and then horizontal reflection in the y-axis.

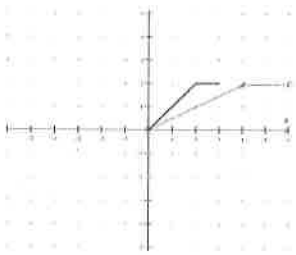
3) vertical stretch BAFO 3, then shift right 2 units and down 11 units.

4) vertical stretch by a factor of 2, then shift right 2 units and down 9 units.

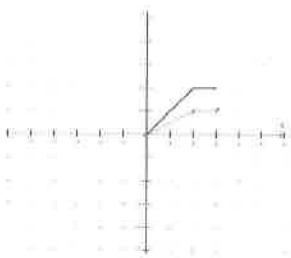
5) a)



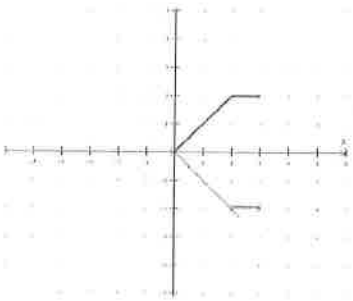
b)



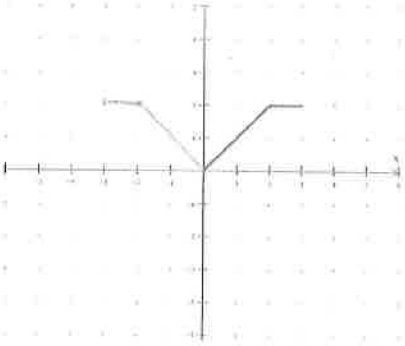
c)



6) a) vertical reflection in the x-axis



b) horizontal reflection in the y-axis



See posted solutions for 7&8

9) $f^{-1}(x) = \left(\frac{x+5}{-2}\right)^2 - 1$

10) $f^{-1}(x) = \pm\sqrt{3(x-2)} + 4$