Chapter 3 Exam Review

SOLUTIONS

MDM4U
Jensen

Using the following data:

24, 25, 25, 25, 26, 27, 29, 30, 32, 32, 32, 32, 34, 34, 35, 36, 38, 39, 41, 43, 44, 45, 46, 47, 48, 49, 51, 54, 55, 57, 58, 60, 65

a) Calculate a bin width that would form six uniform intervals

range = 65 - 24
= 41

bin width = $\frac{42}{6}$
= 7

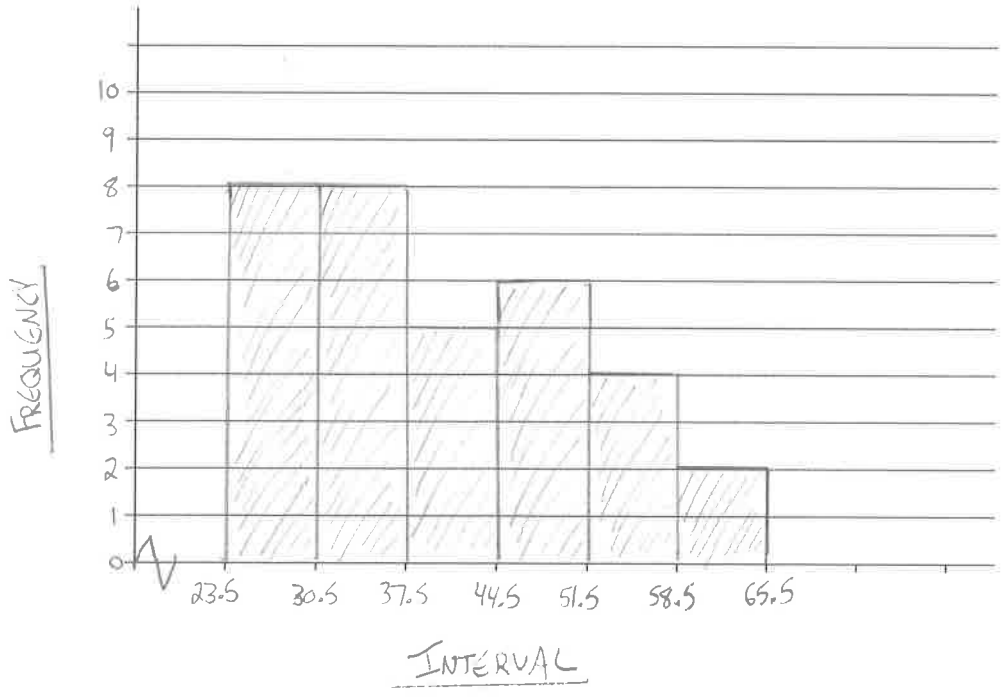
b) Calculate the starting and end point for each of the five intervals. Then complete the frequency distribution.

rounded range up by 1.

starting point = $24 - \frac{1}{2}$
= 23.5

Grade Interval	Frequency
23.5 - 30.5	8
30.5 - 37.5	8
37.5 - 44.5	5
44.5 - 51.5	6
51.5 - 58.5	4
58.5 - 65.5	2

c) Create an appropriate histogram.



2. State the type of distribution that occurs when the mean, median and mode are equal.

Mound Shaped.

3. What type of distribution occurs when the height of each bar is roughly equal?

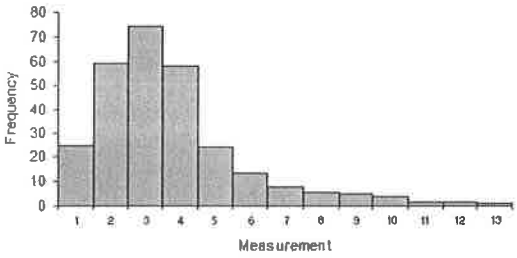
Uniform.

4. What type of distribution occurs when there are peaks at both ends of the range?

U-shaped or Bimodal

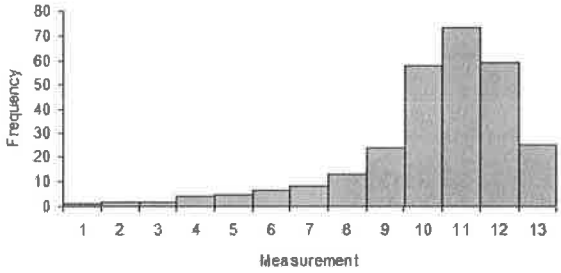
5. What type of distribution is represented in each of the following graphs?

a)



Right Skewed

b)



Left Skewed.

6. The number of patients treated in a dental office on Mondays was recorded for 11 weeks. What are the mean, median, and mode for this set of data by hand.

5, 17, 28, 28, 28, 15, 13, 18, 10, 16, 20

5, 10, 13, 15, 16, (17) 18, 20, 28, 28, 28

$\text{mean} = \frac{\Sigma x}{n}$

$= \frac{198}{11}$

$= 18$

Median = 17

Mode = 28

7. The mean of the values 9, 11, 13, 21, 24, 18, and d is 17. Find d.

$\bar{x} = \frac{\Sigma x}{n}$

$17 = \frac{96+d}{7}$

$119 - 96 = d$

$d = 23$

8. You are taking a class in which your grade is determined from 5 sources: 15% from your homework, 30% from your quizzes (15% per quiz), 25% from your final exam, 15% from your culminating project, and 15% from your speech. Based on the following results, what is the weighted mean of your scores?

Source	Score, x	Weight, w	xw
Homework	80	15	1200
Quiz #1	85	15	1275
Quiz #2	76	15	1140
Project	95	15	1425
Speech	90	15	1350
Final Exam	84	25	2100

Mean: $\bar{x} = \frac{\Sigma xw}{\Sigma w}$

$= \frac{8490}{100}$

$= 84.9\%$

9. The following data represent the salaries of employees at RIM Corporation

Salary (in thousands)	Frequency, f	Midpoint, m	$f \times m$
30-39	18	34.5	621
40-49	15	44.5	667.5
50-59	10	54.5	545
60-69	5	64.5	322.5
70-79	3	74.5	223.5

Mean: $\frac{\Sigma(m \times f)}{\Sigma f} = \frac{2379.5}{51} = 46.66$

10. Each child in a study of infantile autism was given a behavioural test and graded on a scale from 0 (no symptoms) to 116 (maximum severity). The scores of the 21 children in the study were as follows:

27, 35, 65, 67, 47, 46, 63, 44, 34, 51, 17, 40, 41, 60, 24, 48, 29, 73, 60, 41, 47

Use your calculator to calculate the mean and standard deviation of the data.

$\bar{x} = 45.67$

$\sigma = 14.75$

11. The middle 50% of the data surrounding the median is called the Interquartile Range.

12. Listed below are the points scored in the 2009 playoffs for all 20 players on the Stanley Cup winning Pittsburgh Penguins.

0, 1, 1, 2, 4, 4, 5, 5, 6, 7, 7, 9, 9, 13, 14, 14, 14, 15, 31, 36

a) Calculate the mean and standard deviation of the data using your calculator

$\bar{x} = 9.85$

$\sigma = 9.17$

b) Calculate the range of the data

Range = 36 - 0 = 36

c) Find all three quartile values

$Q_1 = 4$

$Q_2 = 7$

$Q_3 = 14$

d) Calculate the interquartile range

$IQR = Q_3 - Q_1$

$= 14 - 4$

$= 10$

13. By hand, find the standard deviation to two decimal places of the ages, in years, of the four Staal brothers playing in the NHL.

29 (Eric), 23 (Jared), 25 (Jordan), 26 (Marc)

$\bar{x} = \frac{29+23+25+26}{4}$

$= 25.75$

$$\sigma = \sqrt{\frac{(29-25.75)^2 + (23-25.75)^2 + (25-25.75)^2 + (26-25.75)^2}{4}}$$

$$\sigma = \sqrt{\frac{18.75}{4}}$$

$$\sigma = 2.165$$

14. The ten test marks of two students are compared.

Sue: 75, 59, 58, 72, 80, 66, 71, 79, 68, 55 Leopold: 90, 83, 55, 84, 72, 63, 50, 65, 52, 91

a) Which student will have the higher average mark?

Sue: $\bar{x} = 68.3$

Leopold will have the highest mark.

Leopold: $\bar{x} = 70.5$

b) Which student gets the more consistent mark? (calculate the standard deviation)

Sue: $\sigma = 8.319$

Sue has the more consistent mark.

Leopold: $\sigma = 14.935$

15. Given the following distribution of mathematics marks on a test out of 25…

Score	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Frequency	1	1	2	0	4	2	3	3	4	6	3	4	4	1	2

Use your calculator to…

a) Calculate the mean score on the test for the class.

$\bar{x} = 18.975$

b) Calculate the standard deviation of the test scores.

$\sigma = 3.525$

16. How many standard deviations is approximately 68% of the data in a normal distribution within?

$\pm 1\sigma$ from the mean

17. How many standard deviations is approximately 95% of the data in a normal distribution within?

$\pm 2\sigma$ from the mean.

18. In a normal distribution, approximately what percent of the data is within 3 standard deviations of the mean?

99.7 %

19. A distribution with a mean of 0 and standard deviation of 1 is called what?

Standard Normal Distribution. $X \sim N(0, 1^2)$

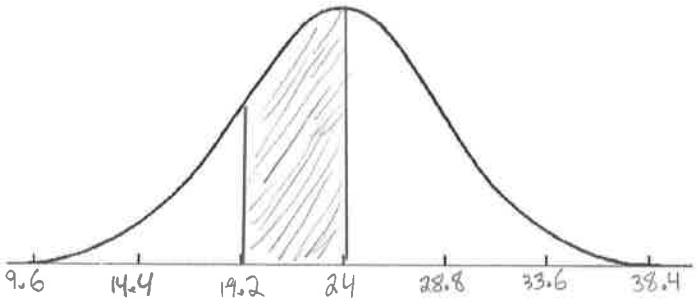
20. The temperatures in Florida for the month of December can be represented by the normal distribution $X \sim N(24, 4.8^2)$

a) What range of temperatures would you expect 68% of the days in December to fall between?

$\bar{x} \pm 1\sigma = 24 \pm 4.8 = 19.2°C$ to $28.8°C$

b) In what percent of the days will the temperature be between 19.2 and 24 degrees Celcius?

% between 19.2 and 24 = 34%

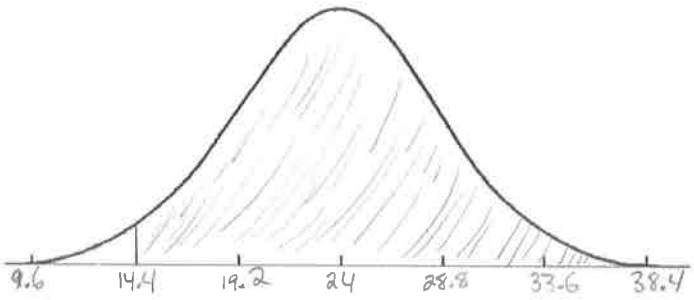


c) In what percent of the days will the temperature be greater than 14.4 degrees Celcius?

% > 14.4 = 13.5 + 34 + 50
= 97.5%

OR

% > 14.4 = 100 - % < 14.4
= 100 - (2.25 + 0.15)
= 100 - 2.4
= 97.6



d) 99.7% of the days will be between what two temperatures?

$\bar{x} \pm 3\sigma = \bar{x} \pm 3(4.8) = 24 \pm 14.4 = 9.6$ to $38.4°C$.

Note: These are approximate answers using the graph of the normal distribution from the textbook. You can use z-scores or your calculator to get exact answers.

21. If $X \sim N(10, 2^2)$, what is the...

a) mean? b) standard deviation? c) variance

$\bar{x} = 10$ $\sigma = 2$ $= \sigma^2 = 2^2 = 4$

22. If the mass of 50 children in Oakville were normally distributed, with a mean of 11.2 kg and a standard deviation of 2.8 kg, determine: $X \sim N(11.2, 2.8^2)$

a) The number of children with a mass less than or equal to 6.1 kg.

Calculator:

$\% \leq 6.1 = \text{normalCDF}(-E99, 6.1, 11.2, 2.8)$
$= 0.03427$

$\# \leq 6.1 = 50(0.03427)$
$= 1.7135$

z-scores

$z_{6.1} = \frac{6.1 - 11.2}{2.8}$
$= -1.82$

or use z-score table.

$\% z \leq -1.82 = \text{normalCDF}(-E99, -1.82)$
$= 0.03438$

$\# \leq 6.1 = 50(0.03438)$
$= 1.719$

b) The percent of children with a mass between 3.7 kg and 18 kg.

$\% 3.7 < X < 18 = \text{normalCDF}(3.7, 18, 11.2, 2.8)$
$= 0.9887$
$= 98.87\%$

z-scores

$z_{3.7} = \frac{3.7 - 11.2}{2.8}$ $z_{18} = \frac{18 - 11.2}{2.8}$
$= -2.68$ $= 2.43$

or use z-score table and the find difference between percentages.

$\% -2.68 < z < 2.43 = \text{normalCDF}(-2.68, 2.43)$
$= 0.9888$
$= 98.88\%$

c) The percent of children with a mass greater than 10.5 kg.

$\% > 10.5 = \text{normalCDF}(10.5, E99, 11.2, 2.8)$
$= 0.5987$
$= 59.87\%$

z-scores

$z_{10.5} = \frac{10.5 - 11.2}{2.8}$
$= -0.25$

$\% z > -0.25 = \text{normalCDF}(-0.25, E99)$
$= 0.5987$

or

$\% z > -0.25 = 1 - z < -0.25$
$= 1 - \text{normalCDF}(-E99, -0.25)$
$= 0.5987$

or use z-score table.

87.2

23. Calculate the z-score, to two decimal place, of $x =$ [redacted] if $\bar{x} = 89.1$ and $\sigma = 3$

$$z_{87.2} = \frac{87.2 - 89.1}{3}$$

$$= -0.63$$

24. Suppose $X \sim N(50, 4^2)$. What value of x would have a z-score of 2.10?

$$z = \frac{x - \bar{x}}{\sigma}$$

$$2.1 = \frac{x - 50}{4}$$

$$x = 58.4$$

25. In $X \sim N(12, 2^2)$, what percent of the data is between 10 and 13? (Use z-scores or calculator)

$\%\ 10 < X < 13 =$ normalCDF$(10, 13, 12, 2)$

$= 0.5328$

$= 53.28\%$

$z_{10} = \frac{10-12}{2} = -1$ $\quad z_{13} = \frac{13-12}{2} = 0.5$

$\%\ -1 < z < 0.5 =$ normalCDF$(-1, 0.5)$

$= 0.5328$

$= 53.28\%$

26. What normal z-score corresponds to the 14th percentile? (use invnormal)

$z =$ invNorm(0.14)

$= -1.08$

27. In $X \sim N(12, 2^2)$, what value of x corresonds to the 87th percentile?

$X =$ invNorm$(0.87, 12, 2)$

$= 14.25$

or

$z =$ invNorm(0.87)

$= 1.13$

$1.13 = \frac{x - 12}{2}$

$x = 14.26$

28. For the distribution $N(16, 3.5^2)$, determine the percent of the data that is within the given interval.

a) $X > 12$

% > 12 = normalCDF(12, E99, 16, 3.5)

= 0.8735

= 87.35

b) $10 < X < 15$

= normalCDF(10, 15, 16, 3.5)

= 0.3443

= 34.43%

c) $X < 18.7$

= normalCDF(-E99, 18.7, 16, 3.5)

= 0.7798

= 77.98%

29. For the normal distribution with a mean of 4 and standard deviation of 0.39, determine the percent of the data that is within the given interval.

a) $1.64 < x < 3.56$

= normalCDF(1.64, 3.56, 4, 0.39)

= 0.1296

= 12.96%

b) $5 < x < 12.8$

= normalCDF(5, 12.8, 4, 0.39)

= 0.0052

= 0.52%

30. A group of students wrote an entrance Math exam and the scores were normally distributed. The mean score was 750 and there was a standard deviation of 95. If Johnny wants to score in the 94th percentile, what score must he get?

x = invNormal(0.94, 750, 95)

= 897.7

31. Mr. Jensen's Average golf score is 80 with a standard deviation of 3. In what percent of his golf rounds will he score less than 72?

= normalCDF(-E99, 72, 80, 3)

= 0.0038

= 0.38%

32. The lengths of nails, in millimeters, at a certain plant are normally distributed with a mean of 20.00 and a standard deviation of 0.21. Nails produced will be rejected unless their lengths are between 19.71 mm and 20.42 mm. What percent of the nails are accepted?

$$\%\ \text{accepted} = \text{normalCDF}(19.71, 20.42, 20, 0.21)$$
$$= 0.8936$$
$$= 89.36\%$$

33. The masses, in grams, of 750 packages of cheese are normally distributed. A package will be rejected if its z-score is -2.57 or less. How many of these packages face rejection?

$$\%\ z < -2.57 = \text{normalCDF}(-E99, -2.57)$$
$$= 0.0051$$
$$= 0.51\%$$

$$\#\ \text{of packages rejected} = 0.51\%\ \text{of}\ 750$$
$$= 0.0051(750)$$
$$= 3.8$$