Chapter 4 Exam Review
MDM4U
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SOLUTIONS

1. An experiment consists of taking one card from a regular 52 card deck. What is the probability that:

a) the card chosen will be a diamond.

$$P(\text{diamond}) = \frac{13}{52} = \frac{1}{4} \text{ or } 0.25$$

b) the card chosen will not be a jack or a king

$$P(\text{not a jack or king}) = 1 - P(\text{jack} \cup \text{king}) = 1 - \left(\frac{4}{52} + \frac{4}{52}\right) = \frac{44}{52} = \frac{11}{13}$$

c) the card chosen will be a diamond or an ace.

$$P(\text{diamond} \cup \text{ace}) = P(\text{diamond}) + P(\text{ace}) - P(\text{diamond} \cap \text{ace})$$
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$
$$= \frac{16}{52} = \frac{4}{13}$$

2. An integer from 1 to 50 inclusive is chosen at random. What is the probability that the integer is:

a) Odd? [0.5]

$$P(\text{odd}) = \frac{25}{50} = \frac{1}{2}$$

b) Divisible by 5 [1/5]

$$P(\text{divisible by 5}) = \frac{10}{50} = \frac{1}{5}$$

c) Not a perfect square? [43/50]

Perfect squares: 1, 4, 9, 16, 25, 36, 49

$$P(\text{perfect square}') = 1 - P(\text{perfect square})$$
$$= 1 - \frac{7}{50}$$
$$= \frac{43}{50}$$

3. Determine the theoretical probability for each of the following events.

a) rolling a 1 on a die $\frac{1}{6}$

b) drawing a face card from a well-shuffled deck $\frac{12}{52} = \frac{3}{13}$

c) drawing a red Queen from a well-shuffled deck $\frac{2}{52} = \frac{1}{26}$

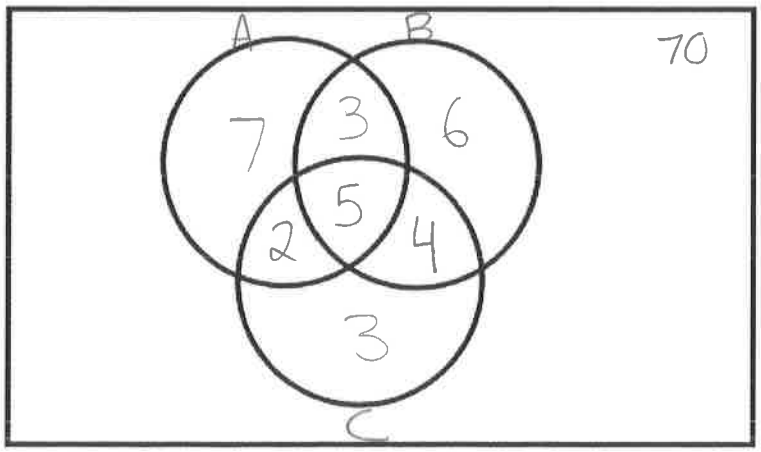
d) rolling a "Q" on a die with each side containing a letter of the English alphabet $\frac{1}{26}$

e) rolling a consonant $\frac{21}{26}$

4. A magazine poll sampling 100 people gives the following results:

17 read magazine A
18 read magazine B
14 read magazine C
8 read magazines A and B
7 read magazines A and C
9 read magazines B and C
5 read all three magazines

a) Illustrate this information on a Venn Diagram:



b) How many of the people polled do not read any of the three magazines?

70

c) How many people read only magazine C?

3

d) What is the probability that one person selected from the sample read magazine A or B?

$\frac{27}{100}$

e) What is the probability that one person selected from the sample read magazine B and C?

$\frac{9}{100}$

5. The probability that a student has a cold is 3/7 and the probability that a student has the flu is 4/9. The probability that a student has both a cold and the flu is 1/3. If a student is picked from the school, at random, determine the probability that the student has a cold or the flu.

$P(\text{cold} \cup \text{flu}) = P(\text{cold}) + P(\text{flu}) - P(\text{cold} \cap \text{flu})$

$= 3/7 + 4/9 - 1/3$

$= 34/63$ or 0.54

6. Find the indicated probability and state whether A and B are mutually exclusive.

a) $P(A) = 0.3, P(B) = 0.4, P(A \cup B) = 0.5, P(A \cap B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.5 = 0.3 + 0.4 - P(A \cap B)$ Not mutually exclusive.

$P(A \cap B) = 0.7 - 0.5$

$P(A \cap B) = 0.2$

b) $P(A) = 0.25, P(B) = 0.3, P(A \cup B) = ?, P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.3 - 0$ Mutually Exclusive.

$= 0.55$

7. Given the two sets below determine each of the following...

$A = \{1, 3, 5, 7, 9\}$ *and* $B = \{2, 7, 9\}$

a) $n(A)$

$= 5$

b) $n(A \cup B)$

$= n(A) + n(B) - n(A \cap B)$

$= 5 + 3 - 2$

$= 6$

c) $n(A \cap B)$

$= 2$

8. If a pair of fair dice is tossed, what is the probability that the sum of the faces showing is even, given that the sum is greater than 7?

$$P(\text{even} \mid > 7) = \frac{n(\text{even} \cap > 7)}{n(>7)}$$

$$= \frac{9}{15}$$

$$= \frac{3}{5} \text{ or } 0.6$$

8: (4,4)(5,3)(3,5)(2,6)(6,2)

9: (6,3)(3,6)(4,5)(5,4)

10: (6,4)(4,6)(5,5)

11: (6,5)(5,6)

12: (6,6)

9. Sally applies to three universities for her post secondary education. If there is an 85% probability that Sally will be accepted into the Engineering program at Waterloo, a 75% probability that she will be accepted to Computer Programming at McMaster and an 88% probability she will be accepted to the University of Ottawa for the Mathematics program, what is the probability that she will be accepted to all three of the programs she has applied for?

$P(\text{accepted to all 3}) = 0.85 \times 0.75 \times 0.88$

$= 0.561$

10. In an archery tournament the probability that Sandy will hit the bullseye is 0.85 and the probability that Adam will hit it is 0.70. Find each of the following:

a) the probability that Adam hits the bullseye and Sandy doesn't

$= 0.7 \times 0.15$

$= 0.105$

b) the probability that neither hits the bullseye

$= 0.15 \times 0.3$

$= 0.045$

11. An experiment consists of drawing three cards one after another with replacement between draws. What is the probability of drawing a spade, a five, and a red card in that order?

$$P(\text{spade}, 5, \text{red}) = P(\text{spade}) \times P(5) \times P(\text{red})$$
$$= \frac{1}{4} \times \frac{1}{13} \times \frac{1}{2}$$
$$= \frac{1}{104}$$

12. Two traffic lights operate independently of each other. the probability of being stopped at the first light is 0.2 and the probability of being stopped at the second is 0.6. Find the probability of each of the following:

a) you will stop at both lights $P(\text{stop}, \text{stop}) = 0.2 \times 0.6$
$= 0.12$

b) you will go through both lights $P(\text{go}, \text{go}) = 0.8 \times 0.4$
$= 0.32$

c) you will be stopped at the first but not the second light.

$$P(\text{stop}, \text{go}) = 0.2 \times 0.4$$
$$= 0.08$$

13. A bag contains 2 red, 4 white and 6 black jelly beans.

a) One jelly bean is drawn at random. What is the probability of drawing a red jelly bean?

$$P(\text{red}) = \frac{2}{12} = \frac{1}{6}$$

b) One jelly bean is drawn and then replaced. A second jelly bean is drawn. What is the probability of drawing a red bean and then a black one?

$$P(\text{red}, \text{black}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

c) One jelly bean is drawn but not replaced. Then a second jelly bean is drawn. What is the probability of drawing a red jelly bean and then a black one?

$$P(\text{red} \cap \text{black}) = P(\text{red}) \times P(\text{black} \mid \text{red})$$
$$= \frac{1}{6} \times \frac{6}{11}$$
$$= \frac{1}{11}$$

14. The probability that Crosby gets a goal in a game is 0.35 and the probability that he gets an assist in a game is 0.8. The probability that Crosby gets a goal and an assist in a game is 0.25. What is the probability that Crosby gets an assist given that he has already gotten a goal in a game.

$$P(\text{assist} \mid \text{goal}) = \frac{P(\text{assist} \cap \text{goal})}{P(\text{goal})} = \frac{0.25}{0.35} = \frac{5}{7} \text{ or } 0.714$$

15. A survey at a school asked students if they were ill with a cold or the flu during the last month. The results were as follows. None of the students had both a cold and the flu.

179

	Cold	Flu	Healthy
Females	32	18	47
Males	25	19	38

Use the results to find...

a) $P(cold) = \frac{57}{179}$

b) $P(healthy \mid female) = \frac{n(\text{healthy} \cap \text{female})}{n(\text{female})} = \frac{47}{97}$

c) $P(male \mid flu) = \frac{n(\text{male} \cap \text{flu})}{n(\text{flu})} = \frac{19}{37}$

d) $P(cold \mid male) = \frac{n(\text{cold} \cap \text{male})}{n(\text{male})} = \frac{25}{82}$

16. There are 3 twenty dollar bills, 6 ten dollar bills and 9 five dollar bills in a box. Suppose a game is played in which a bill is randomly taken from the box, replaced, and then a second bill is drawn from the box. If you are allowed to keep the second bill only if it was the same type as the one that was drawn the first time, calculate the probability that you will be able to keep any bill.

$P(\text{keep \$20}) = \left(\frac{3}{18}\right)\left(\frac{3}{18}\right) = \frac{1}{36}$

$P(\text{keep \$10}) = \left(\frac{6}{18}\right)\left(\frac{6}{18}\right) = \frac{1}{9}$

$P(\text{keep \$5}) = \left(\frac{9}{18}\right)\left(\frac{9}{18}\right) = \frac{1}{4}$

$P(\text{keep any}) = P(\text{keep 20}) + P(\text{keep 10}) + P(\text{keep 5})$

$= \frac{1}{36} + \frac{1}{9} + \frac{1}{4}$

$= \frac{1}{36} + \frac{4}{36} + \frac{9}{36}$

$= \frac{14}{36}$

$= \frac{7}{18}$

17. A golfer has 2 pairs of shoes, 7 belts, 7 shirts, 4 pants, and 2 hats. How many different outfits can he make?

$n(\text{outfits}) = 2 \times 7 \times 7 \times 4 \times 2$

$= 784$

18. How many 4-digit numbers can be made from the digits 1 to 9 if:

a) Repetition of digits is not allowed?

$P(9,4) = \frac{9!}{5!} = 9 \times 8 \times 7 \times 6 = 3024$

b) Repetition is allowed and the number must be less than 5000?

$= 4 \times 9 \times 9 \times 9$

$= 2916$

19. In how many ways can the letters of the word "accuracy" be arranged if:

a) There are no restrictions?

$= \frac{8!}{2!3!} = \frac{40320}{12} = 3360$

b) The arrangement must end with a "y"?

$= \frac{7!}{2!3!} = \frac{5040}{12} = 420$

20. In how many ways can 6 friends be seated in a row at the movies?

$6! = 720$ or $P(6,6) = \frac{6!}{0!} = \frac{720}{1} = 720$

21. How many arrangements are there for the letters in the word GOALIE ?

$= 6!$

$= 720$

22. All 12 members of a Student Parliament had their picture taken.

a) In how many ways can the 12 pictures be hung in a row outside the student parliament office?

$= 12!$

$= 479\,001\,600$

b) In how many ways can 5 of the 12 pictures be hung in a row?

$P(12,5) = \frac{12!}{7!} = 12 \times 11 \times 10 \times 9 \times 8 = 95040$

c) In how many ways can 7 of the 12 pictures be hung in a row if Patrick's picture must be first?

$P(11,6) = \frac{11!}{5!} = 11 \times 10 \times 9 \times 8 \times 7 \times 6 = 332640$

d) In how many ways can all 12 pictures be hung if Lisa and Vince's pictures must be hung beside each other?

$11! \times 2!$

$= 79\,833\,600$

23. There are 10 males and 18 females in the Data Management class. How many different committees of 5 students can be formed if:

a) There are no restrictions?

$= \binom{28}{5}$

$= 98280$

b) There must be 3 males and 2 females?

$= \binom{10}{3}\binom{18}{2}$

$= 120(153)$

$= 18360$

c) Jessica and Eric must be on the committee?

$= \binom{26}{3}$

$= 2600$

d) There must be a chair, co-chair, secretary, treasurer and speaker?

$= P(28,5)$

$= \frac{28!}{23!}$

$= 28 \times 27 \times 26 \times 25 \times 24$

$= 11793600$

24. A 5-letter/digit computer password is given to all the employees of RIM Corporation. How many different passwords can be formed using any of the 26 letters of the alphabet and the 10 numerical digits if no repetition is allowed. Only lower case letters are used.

$= P(36,5)$

$= \frac{36!}{31!}$

$= 36 \times 35 \times 34 \times 33 \times 32$

$= 45\,239\,040$

25. In how many ways can 4 people be selected from a group of 15 to work on a committee?

$= C(15,4)$

$= 1365$

26. Mr. Math is to answer any 8 out of 10 questions on an examination.

a) How many different groups of 8 questions can Mr. Math choose?

$= C(10,8)$

$= 45$

b) How many ways can Mr. Math choose the questions if he must answer at least 4 of the first 5 questions?

$= \binom{5}{4}\binom{5}{4} + \binom{5}{5}\binom{5}{3}$

$= 5(5) + 1(10)$

$= 35$

27. A committee of 4 is to be chosen from a club which has a membership of 10 men and 12 women. How many ways can the committee be formed if:

a) it is to contain at least 2 women? [

$= \binom{12}{2}\binom{10}{2} + \binom{12}{3}\binom{10}{1} + \binom{12}{4}\binom{10}{0}$

$= 66(45) + 220(10) + 495(1)$

$= 5665$

b) any number of men and women may be selected but Mr. and Ms. Math refuse to serve together? = total # of committees − # of committees they do serve together

$= \binom{22}{4} - \binom{20}{2}$

$= 7315 - 190$

$= 7125$

28. The Prom committee is to be selected from graduating students at KCC. The committee is to consist of five students selected randomly from a list of 6 males and 8 females. What is the probability that:

a) The first 2 students selected are female?

$= \left(\frac{8}{14}\right)\left(\frac{7}{13}\right)$

$= \frac{4}{13}$

b) Three of the five students are male?

$$\frac{\binom{6}{3}\binom{8}{2}}{\binom{14}{5}} = \frac{20(28)}{2002} = \frac{40}{143} \text{ or } 0.28$$

29. A committee of 6 is to be chosen from the 28 students in a class. If there are 10 males and 18 females in the class, in how many ways can this be done if there must be at least three females on the committee?

$= \binom{18}{3}\binom{10}{3} + \binom{18}{4}\binom{10}{2} + \binom{18}{5}\binom{10}{1} + \binom{18}{6}\binom{10}{0}$

$= 816(120) + 3060(45) + 8568(10) + 18564$

$= 339864$

30. How many poker hands (5 cards) contain:

a) exactly 3 aces? **[4512]**

$= \binom{4}{3}\binom{48}{2}$

$= 4512$

b) three of a kind? **[58656]**

$= \binom{13}{1}\binom{4}{3}\binom{48}{2}$

$= 58656$

c) exactly 1 ace and 1 king? **[211904]**

$= \binom{4}{1}\binom{4}{1}\binom{44}{3}$

$= 211904$

31. In how many ways can a committee of 3 men and 2 women be chosen from 7 men and 3 women?

$= \binom{7}{3}\binom{3}{2}$

$= 35(3)$

$= 105$