Unit 1 Exam Review

Chapter 1: Linear Systems

Chapter 2: Analytic Geometry

CHAPTER 1

Linear System: two or more linear equations that are considered at the same time.

To solve a linear system means to find the **Point of Intersection**

How to Solve a Linear System:

Method of Substitution: solving a linear system by substituting for one variable from one equation into the other equation.

Method of Elimination: solving a linear system by adding or subtracting to eliminate one of the variables.

Steps to Substitution

1. Isolate a variable (choose the easiest one).
2. Substitute this expression into the second equation.
3. Combine like terms (remember order of operations).
4. Solve for the variable.
5. Substitute that value into one of the original equations and solve for the second variable.

Solve the following linear system using substitution:

$$5 = 2y - x$$
$$7 = 3y - 2x$$

1) isolate a variable
$$5 = 2y - x$$
$$x + 5 = 2y$$
$$x = 2y - 5$$

2) $7 = 3y - 2(2y - 5)$
$$7 = 3y - 4y + 10$$
$$7 - 10 = -y$$
$$-3 = -y$$
$$y = 3$$

$$5 = 2(3) - x$$
$$5 = 6 - x$$
$$x = 6 - 5$$
$$x = 1$$

The POI is $(1, 3)$

Steps for Elimination

1. Get both equations into the form of $x + y = \#$
2. Determine how to get rid of one of the variables.
3. Add OR Subtract like terms in the equation to eliminate the chosen variable.
4. Solve the resulting equation for the remaining variable.
5. Substitute that value into one of the original equations to solve for the second variable.

Solve the linear system by elimination:

$$5x + 2y = -11$$
$$3x + 2y = -9 \quad -$$

$$2x = -2$$
$$x = -1$$

Plug $x = -1$ into either equation

$$5(-1) + 2y = -11$$
$$-5 + 2y = -11$$
$$2y = -6$$
$$y = -3$$

POI is $(-1, -3)$

Solve the linear system using elimination

$$10x + 4y = -1$$
$$8x - 2y = 7$$

$$10 + 4y = -1$$
$$16x - 4y = 14 \quad +$$

$$26x + 0y = 13$$
$$26x = 13$$
$$x = \frac{13}{26}$$
$$x = \frac{1}{2}$$

Plug $x = \frac{1}{2}$ into the first eq.

$$10\left(\frac{1}{2}\right) + 4y = -1$$
$$5 + 4y = -1$$
$$4y = -6$$
$$y = -\frac{3}{2}$$

POI is $\left(\frac{1}{2}, -\frac{3}{2}\right)$

A small store sells used CDs and DVDs. The CDs sell for $9/each. The DVDs sell for $11/each. Cody is working part time and sells a total of $204 worth of CDs and DVDs during his shift. He knows that 20 items were sold. How many CDs did Cody sell? How many DVDs did Cody sell?

$$CD_S = X$$
$$DVD_S = Y$$

$$9x + 11y = 204$$
$$x + y = 20 \quad \text{(×9)}$$

$$9x + 11y = 204$$
$$9x + 9y = 180 \quad -$$
$$\overline{0x + 2y = 24}$$
$$y = 12$$

$$x + 12 = 20$$
$$x = 8$$

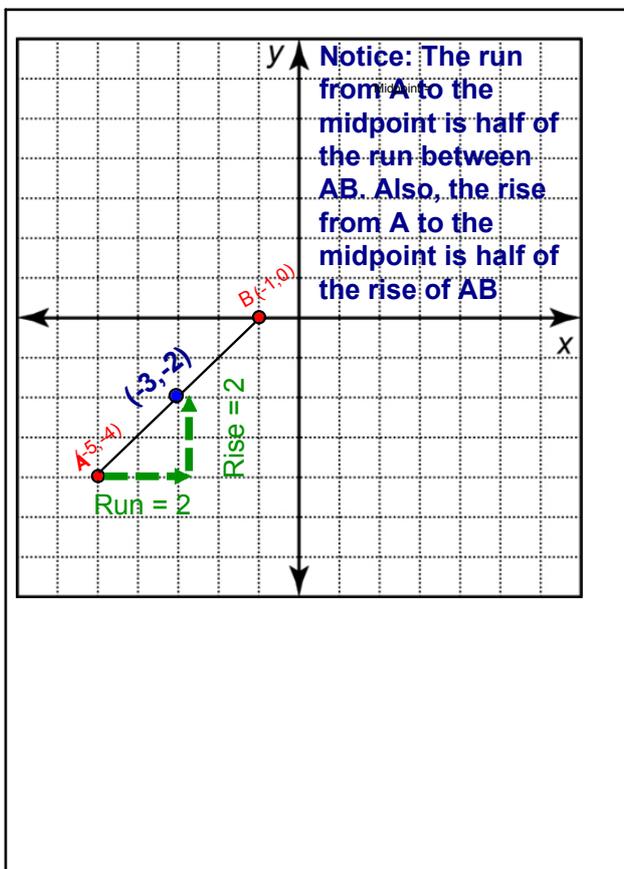He sold 8 CD's and 12 DVD'S

CHAPTER 2

Midpoint: point that divides a line segment into two equal line segments

Length: distance between two points

Median: Line segment joining a vertex of a triangle to the midpoint of the opposite side

Right Bisector: the line that passes through the midpoint of a line segment and intersects it at a 90 degree angle.

Circle: is the set of all points that are the same distance from a fixed point, the center.

Notice: The run from A to the midpoint is half of the run between AB. Also, the rise from A to the midpoint is half of the rise of AB

B(-1,0)
(-3,-2)
A(-5,-4)
Rise = 2
Run = 2

Find the midpoint of the line segment joined by the following endpoints:

$$\text{MIDPOINT} = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

$$x_1 y_1, \ x_2 y_2$$

1) A(3,9) B(5,11)

$$\text{Mid } AB = \left(\frac{3+5}{2} , \frac{9+11}{2} \right)$$
$$= (4, 10)$$

2) A(5,1) B(6,0)

$$\text{Mid } AB = \left(\frac{5+6}{2} , \frac{1+0}{2} \right)$$
$$= \left(\frac{11}{2} , \frac{1}{2} \right)$$

Pythagorean Theorem:
$$a^2 + b^2 = c^2$$
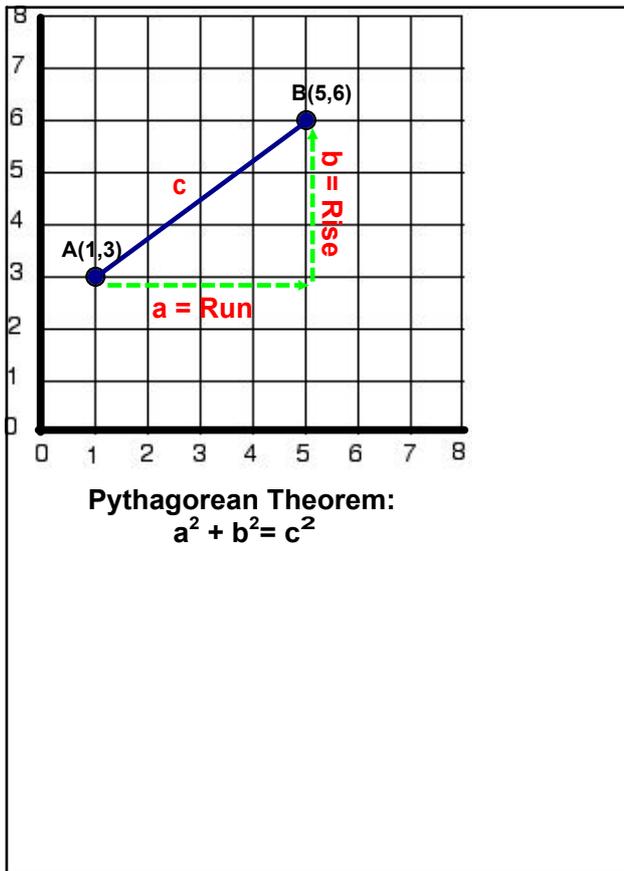
$$\text{Length} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Find the length of the line with endpoints A(-3,1) and B(4,5)
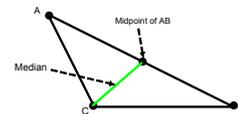
$x_1 \; y_1 \qquad x_2 \; y_2$

$$\text{Length}_{AB} = \sqrt{(4--3)^2 + (5-1)^2}$$
$$= \sqrt{7^2 + 4^2}$$
$$= \sqrt{49+16}$$
$$= \sqrt{65}$$
$$= 8.1$$

$$\text{Length} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Find the length of the line with endpoints A(5,7) and B(1,-1)

$x_1 \; y_1 \qquad x_2 \; y_2$

$$\text{Length} = \sqrt{(1-5)^2 + (-1-7)^2}$$
$$= \sqrt{(-4)^2 + (-8)^2}$$
$$= \sqrt{16+64}$$
$$= \sqrt{80}$$
$$= 8.9$$

How do we find the equation of the median from a vertex?

1) Determine the coordinates of the midpoint of the side that is opposite the vertex

2) Find the slope of the line that connects the vertex to the midpoint of the opposite side

3) Using the coordinates of the vertex (or the midpoint of the opposite side) and the slope of the line, solve for the y-intercept of this line.

4) Write the equation of the line that connects the vertex to the midpoint of the opposite side in **y=mx+b** format. Plug in your values for 'm' and 'b'.

Determine the equation for the <u>median</u> from vertex A for the triangle with vertices A(4,4), B(-6,2), and C(2,0)

A(4,4)

B(-6,2)   D(-2,1)   (2,0)
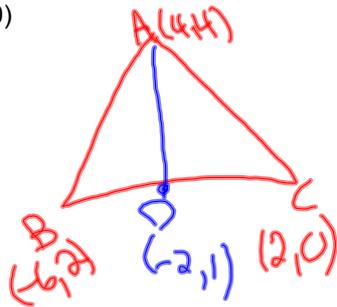
1) Mid of BC

$$= \left(\frac{-6+2}{2}, \frac{2+0}{2} \right)$$

$$= (-2,1)$$

2) slope of AD

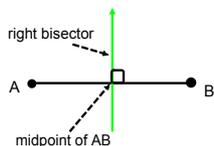$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-4}{-2-4}$$

$$= \frac{-3}{-6}$$

$$= \frac{1}{2}$$

3) Find y-int.

$m = \frac{1}{2}$
D is $(-2,1)$ Plug into $y=mx+b$

                    x   y
$$1 = \frac{1}{2}(-2) + b$$

$$1 = -1 + b$$

$$2 = b$$

$$\boxed{y = \frac{1}{2}x + 2}$$

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right bisector

A •————□————• B

midpoint of AB

How to find the equation of the right bisector of line segment AB:

1) Find the slope of line AB
2) Find the perpendicular slope of AB
3) Find the midpoint of AB
4) Use the perpendicular slope and the midpoint of AB to write the equation of the line that bisects AB and is perpendicular to it (write it in y=mx+b form)

Find the equation of the right bisector of the line segment with endpoints A(1,4) and B(3,-2)

(1,4)   (3,-2)
  A   B

1) $m_{AB} = \frac{-2-4}{3-1}$

$$= \frac{-6}{2}$$

$$= -3$$

2) $\perp m = \frac{1}{3}$

3) mid of AB

$$= \left(\frac{1+3}{2}, \frac{4+-2}{2} \right)$$

$$= (2, 1)$$

4) Find y-int.

$(2,1)$ plug into
 x y $y = mx + b$
$m = \frac{1}{3}$

$$1 = \frac{1}{3}(2) + b$$

$$\frac{3}{3} = \frac{2}{3} + b$$

$$\frac{1}{3} = b$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

A **circle** is the set of all points that are the same distance from a fixed point, the center.

The **radius** is the distance from the center of the circle to any point on the circle.

> Equation of a Circle:
>
> $r^2 = (x)^2 + (y)^2$

Write the equation of a circle with center (0,0) and a radius of 3 units

$r = 3$

$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = 3^2$$
$$x^2 + y^2 = 9$$

What is the radius of a circle defined by the equation $x^2 + y^2 = 81$

$$r^2 = 81$$

$$r = \sqrt{81}$$

$$r = 9$$

A circle has a center (0,0) and passes through the point (8,-6). Find the equation of the circle

 x y

$$x^2 + y^2 = r^2$$

$$8^2 + (-6)^2 = r^2$$

$$64 + 36 = r^2$$

$$100 = r^2$$

$$\boxed{x^2 + y^2 = 100}$$

Is the point B(-5,9) inside this circle $x^2 + y^2 = 100$

xy

$(-5)^2 + (9)^2 = 100$

$25 + 81 = 100$

$106 \neq 100$

$106 > 100$

So outside the circle