L1 – 3.4 Solve Rational Equations and Inequalities
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Part 1: Rational Expressions

Rational Expression: the quotient of two polynomials, $\frac{p(x)}{q(x)}$, where $q(x) \neq 0$

Example 1: Simplify and state the restrictions of each rational expression

a) $\frac{2x^2-8}{x^2+3x+2}$

b) $\frac{x^3-x^2-x+1}{3x^3-3}$

Part 2: Solve Rational Equations

Steps for solving rational equations:

1) Fully factor both sides of the equation
2) Multiply both sides by a common denominator (cross multiply if appropriate)
3) Continue to solve as you would a normal polynomial equation
4) State restrictions throughout (values of x that would make denominator equal zero)

Example 2: Solve each equation

a) $\frac{4}{3x-5} = 4$

b) $\frac{6}{x-2} = x - 1$

c) $\frac{x-5}{x^2-3x-4} = \frac{3x+2}{x^2-1}$

Part 3: Solve Rational Inequalities

REMEMBER: Solving________________ is the same as solving ____________. However, when both sides of an inequality are multiplied or divided by a ____________ number, the inequality sign must be ____________.

Steps for solving rational inequalities algebraically:

1) Use inverse operations to move all terms to one side of the inequality
2) Combine the expressions on the using a common denominator
3) Factor the polynomial
4) Find the interval(s) where the function is positive or negative by making a factor table

To make a factor table:

- Use x-intercepts and vertical asymptotes to divide in to intervals
- Use a test point within each interval to find the sign of each factor
- Determine the overall sign of the product by multiplying signs of each factor within each interval.

Example 3: Solve each inequality algebraically

a) $\frac{x^2+6x+5}{2x^2-7x+3} < 0$

b) $x - 2 < \frac{8}{x}$

c) $\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$

d) $\frac{x^3+6x^2-2x}{x^2+4} \geq 2$