**L1 – 6.3 Transformations of Exponential and Logarithmic Functions**
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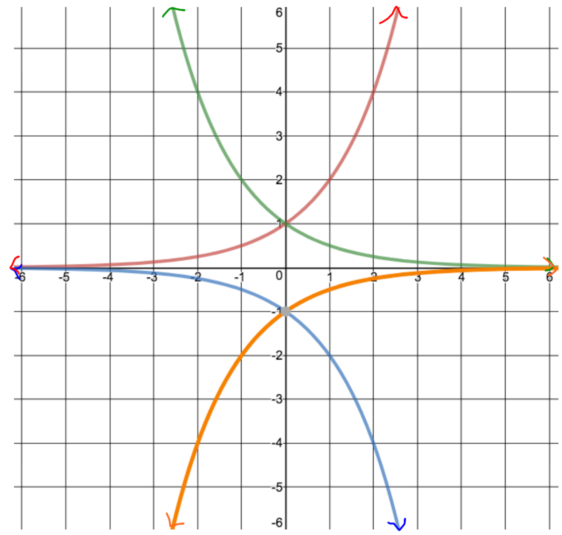
## Part 1: Properties of Exponential Functions

**General Equation:** $y = a(b)^{k(x-d)} + c$ where the base function is $y = b^x$

There are 4 possible shapes for an exponential function

1) $a > 0$ and $b > 1$ (ex. $y = 2^x$)
2) $a > 0$ and $0 < b < 1$ (ex. $y = \left(\frac{1}{2}\right)^x$)
3) $a < 0$ and $b > 1$ (ex. $y = -1(2)^x$)
4) $a < 0$ and $0 < b < 1$ (ex. $y = -1\left(\frac{1}{2}\right)^x$)



To graph the base function $y = b^x$, Find the following key features:

- Horizontal asymptote
  - Starts at $y = 0$ and can be shifted by $c$
- $y - intercept$
  - set $x = 0$ and solve
- At least one other point to be sure of shape
  - Common to choose $x = 1$ and solve for $y$

You can then use transformational properties of $a$, $k$, $d$, and $c$ to graph a transformed function

## Part 2: Transformations of Exponential Functions

**Example 1:** Sketch the graph of $f(x) = 2(3)^{x+4} - 5$ and $g(x) = -3^{\frac{1}{2}x} + 4$ using transformations

| $y = 3^x$ | |
|---|---|
| $x$ | $y$ |
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| | |
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| $f(x) = 2(3)^{x+4} - 5$ | |
|---|---|
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| $g(x) = -3^{\frac{1}{2}x} + 4$ | |
|---|---|
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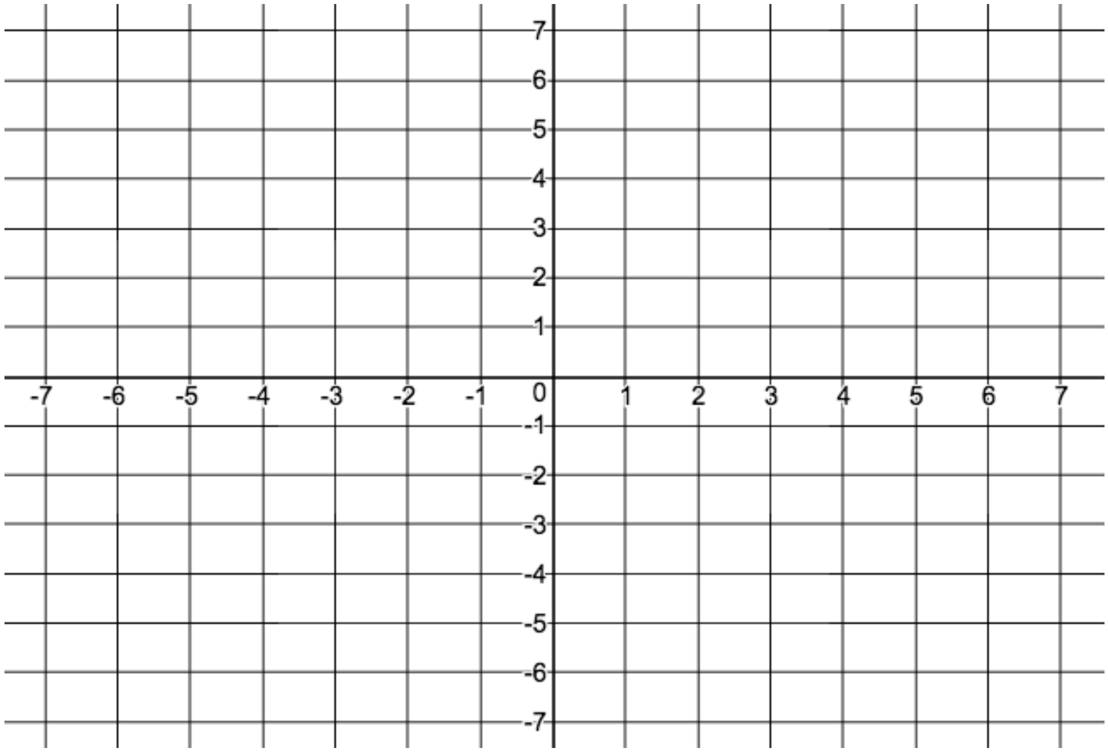


## Part 3: Properties of Logarithmic Functions

**General Equation:** $y = a\log_b[k(x-d)] + c$ where the base function is $y = \log_b x$

Remember that $y = \log_b x$ is the inverse of the exponential function $y = b^x$

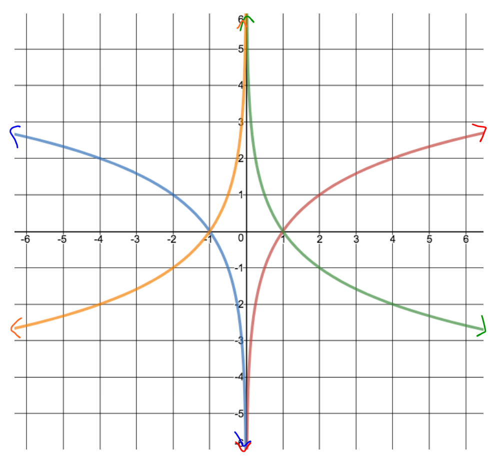
There are 4 possible shapes for a logarithmic function

**1)** $k > 0$ and $b > 1$ (ex. $y = \log_2(x)$)

**2)** $k > 0$ and $0 < b < 1$ (ex. $y = \log_{0.5}(x)$)

**3)** $k < 0$ and $b > 1$ (ex. $y = \log_2(-x)$)

**4)** $k < 0$ and $0 < b < 1$ (ex. $y = \log_{0.5}(-x)$)



To graph the base function $y = \log_b x$, Find the following key features:

- Vertical asymptote
  - Starts at $x = 0$ and can be shifted by $d$
- $x - intercept$
  - set $y = 0$ and solve
- At least one other point to be sure of shape
  - Common to choose $y = 1$ and solve for $x$

**Part 4: Transformations of Logarithmic Functions**

**Example 2:** Sketch the graph of $f(x) = -4\log_3(x) + 2$ and $g(x) = \log_3[-(x+2)] - 4$ using transformations

| $y = \log_3(x)$ | |
|---|---|
| $x$ | $y$ |
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| $f(x) = -4\log_3(x) + 2$ | |
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| $g(x) = \log_3[-(x+2)] - 4$ | |
|---|---|
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