L1 – Increasing / Decreasing Unit 2
MCV4U
Jensen

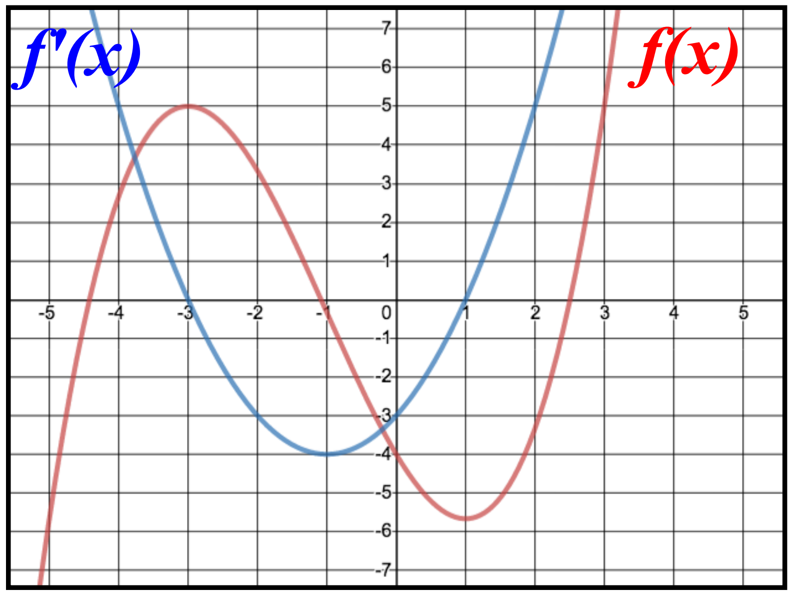
Increasing: As x-values increase, y-values are increasing

Decreasing: As x-values increase, y-values are decreasing

Part 1: Discovery

$f(x) = \frac{1}{3}x^3 + x^2 - 3x - 4$

$f'(x) = x^2 + 2x - 3$



a) Over which values of x is $f(x)$ increasing?

$x < -3$ and $x > 1$

b) Over which values of x is $f(x)$ decreasing?

$-3 < x < 1$

c) What is true about the graph of $f'(x)$ when $f(x)$ is increasing?

$f'(x) > 0$; it is above the x-axis

d) What is true about the graph of $f'(x)$ when $f(x)$ is decreasing?

$f'(x) < 0$; it is below the x-axis

Effects of $f'(x)$ on $f(x)$: When the graph of $f'(x)$ is positive, or above the x-axis, on an interval, then the function $f(x)$ **increases** over that interval. Similarly, when the graph of $f'(x)$ is negative, or below the x-axis, on an interval, then the function $f(x)$ **decreases** over that interval.

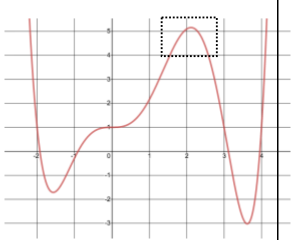
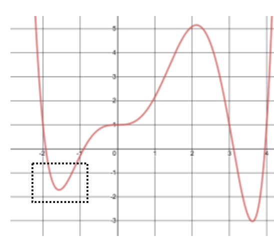
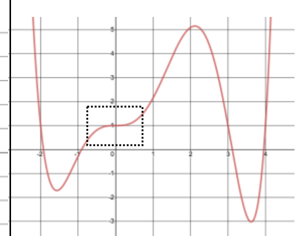
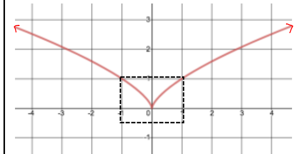
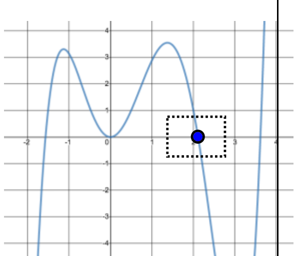
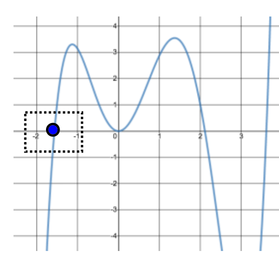
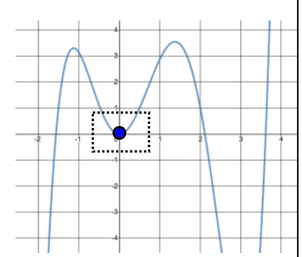
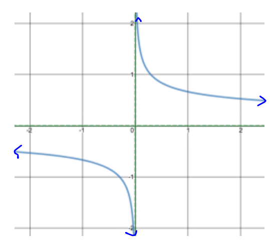
If $f'(x) > 0$ on an interval, $f(x)$ is increasing on that interval

If $f'(x) < 0$ on an interval, $f(x)$ is decreasing on that interval

Part 2: Properties of graphs of $f(x)$ and $f'(x)$

A critical number is a value $'a'$ in the domain where $f'(a) = 0$ or $f'(a)$ does not exist.

A critical number could yield…

	A local max	A local min	Neither	Max/Min at Cusp
$f(x)$				
$f'(x)$				
	$f(x)$ has a local max $f'(x) = 0$ and changes from + to -	$f(x)$ has a local min $f'(x) = 0$ and changes from - to +	No local max/min $f'(x) = 0$ but does not change signs	$f(x)$ has a local min $f'(x)$ does not exist but changes from – to +

Conclusion:

Local extrema occur when the sign of the derivative CHANGES. If the sign of the derivative does not change, you do not have a local extrema.

A critical number of a function is a value of a in the domain of the function where either $f'(a) = 0$ or $f'(a)$ does not exist. If a is a critical number, $(a, f(a))$ is a critical point. Critical points could be local extrema but not necessarily. You must test around the critical points to see if the derivative changes sign. This is called the **'First Derivative Test'**.

Example 1: Determine all local extrema for the function below using critical numbers and the first derivative test. State when the function is increasing or decreasing.

$f(x) = 2x^3 - 9x^2 - 24x - 10$

$f'(x) = 6x^2 - 18x - 24$

$0 = 6(x^2 - 3x - 4)$

$0 = 6(x - 4)(x + 1)$

$x_1 = 4 \quad x_2 = -1$

Critical Numbers: $x_1 = 4 \quad x_2 = -1$

Critical Points: $(4, -122)$ and $(-1, 3)$

$f(4) = 2(4)^3 - 9(4)^2 - 24(4) - 10 = -122$

$f(-1) = 2(-1)^3 - 9(-1)^2 - 24(-1) - 10 = 3$

Sign Chart:

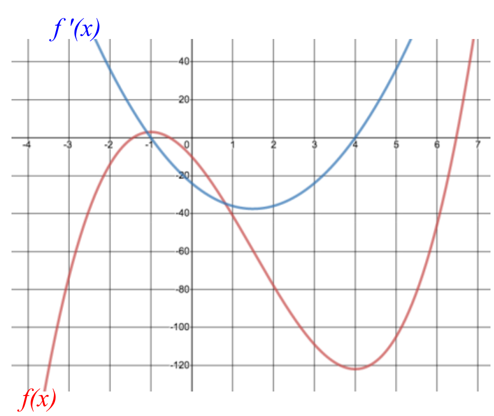
	$-\infty$ to -1	-1 to 4	4 to ∞
Test value for x	-2	0	5
$f'(x)$	$+$	$-$	$+$
$f(x)$	Increasing ↗	Decreasing ↘	Increasing ↗

Local max at $(-1, 3)$

Local min at $(4, -122)$

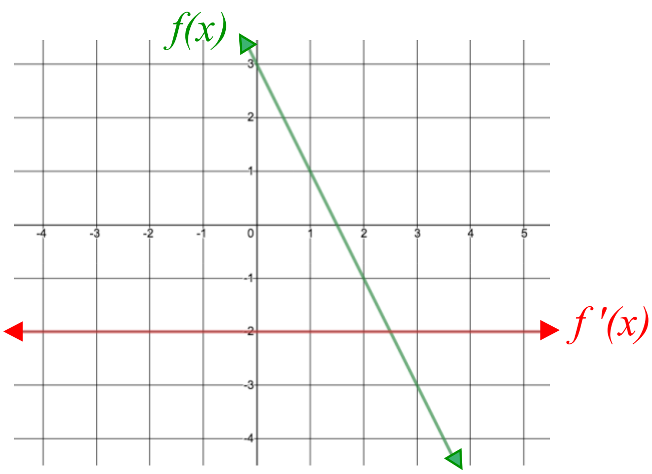
Increasing: $x < -1$ or $x > 4$
Decreasing: $-1 < x < 4$

Notice how we could use the graph of the derivative to verify our solution:



Example 2: For each function, use the graph of $f'(x)$ to sketch a possible function $f(x)$.

a)

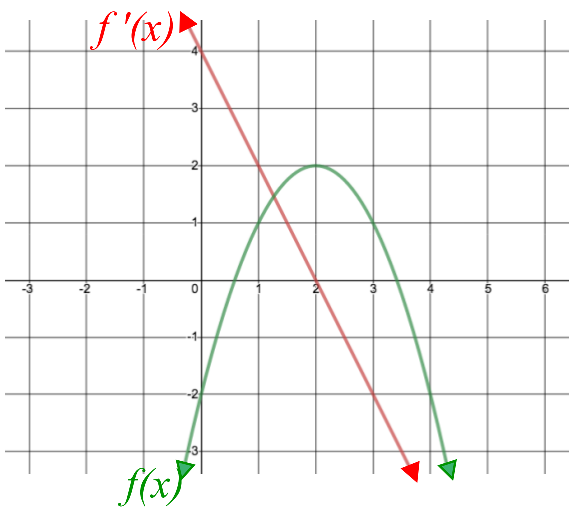


$f'(x)$ is the constant function $y = -2$.

Therefore, $f(x)$ must be a linear function with a slope of -2

The y-intercept could be anywhere.

b)



$f'(x)$ is a linear function

Therefore, $f(x)$ must be a quadratic function

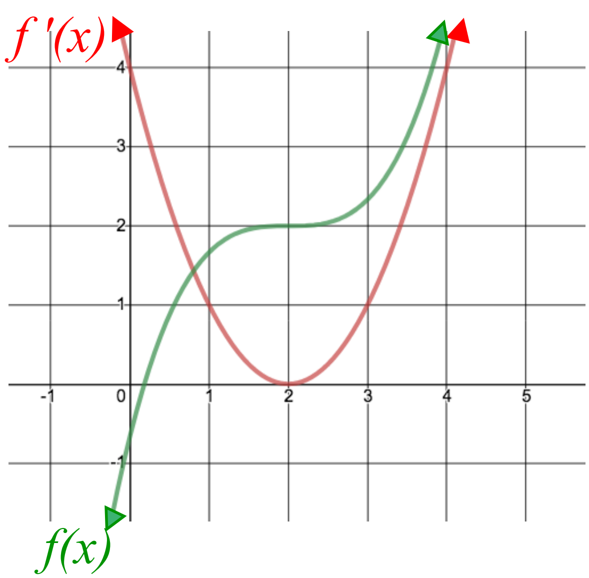
$f'(x) > 0$ when $x < 2$ and $f'(x) < 0$ when $x > 2$

Therefore, $f(x)$ is increasing when $x < 2$ and decreasing when $x > 2$

$f'(x)$ switching from + to – at $x = 2$

There must be a local max at $f(2)$.

c)



$f'(x)$ is a quadratic function

Therefore, $f(x)$ must be a cubic function

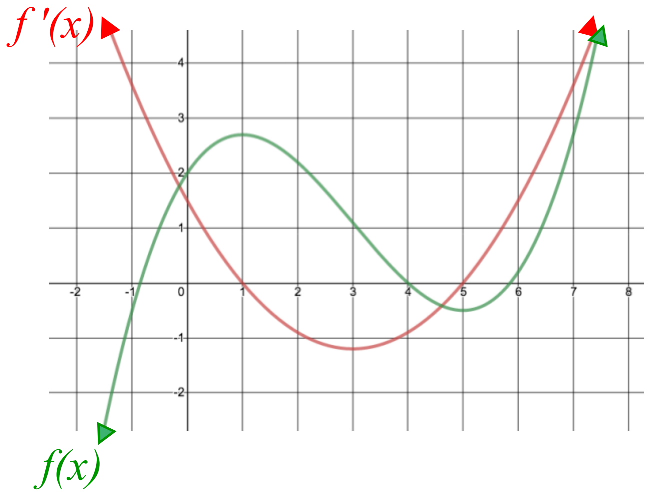
$f'(x) > 0$ when $x < 2$ when $x > 2$

Therefore, $f(x)$ is increasing when $x < 2$ and when $x > 2$

$f'(x)$ does not switch signs on either side of $x = 2$

Therefore, there is no local min or max at $f(2)$ but the tangent line would be horizontal at that point.

d)



$f'(x)$ is a quadratic function

Therefore, $f(x)$ must be a cubic function

$f'(x) > 0$ when $x < 1$ when $x > 5$

Therefore, $f(x)$ is increasing when $x < 1$ and when $x > 5$

$f'(x) < 0$ when $1 < x < 5$

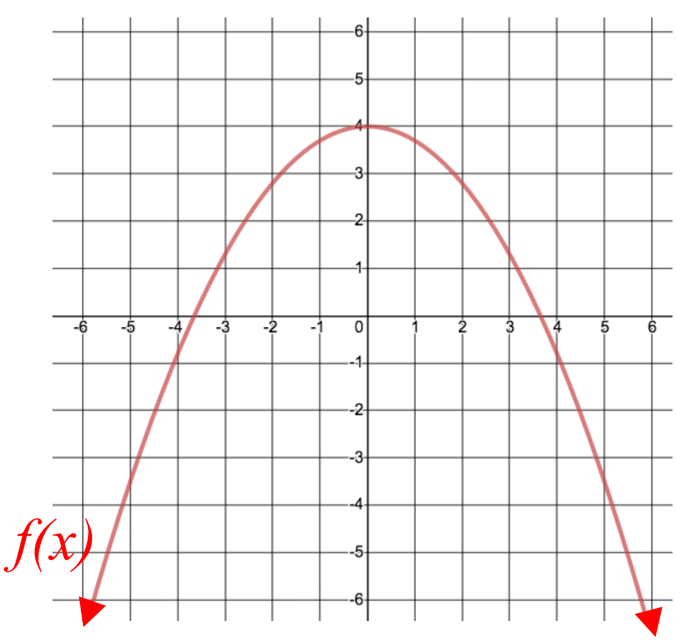
Therefore, $f(x)$ is decreasing when $1 < x < 5$

$f'(x)$ switches signs on either side of $x = 1$ (from + to -) and $x = 5$ (from – to +)

Therefore, there is a max at $f(1)$ and a local min at $f(5)$

Example 3: Sketch a continuous function for each set of conditions

a) $f'(x) > 0$ when $x < 0$, $f'(x) < 0$ when $x > 0$, $f(0) = 4$

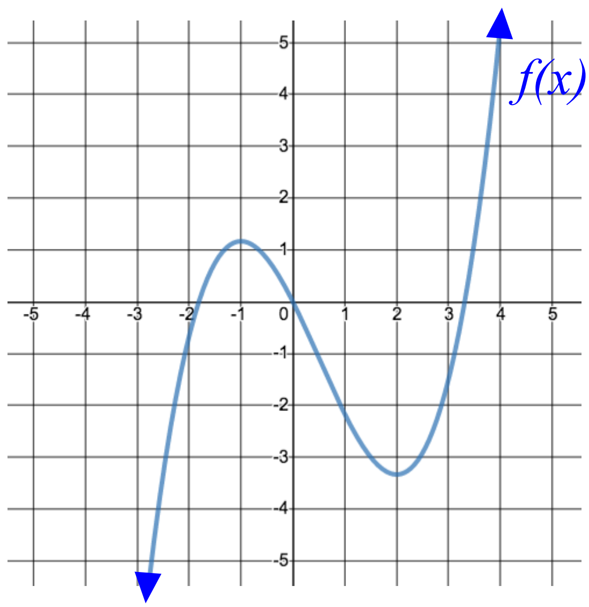


$f(x)$ is increasing when $x < 0$ and decreasing when $x > 0$.

Therefore, there must be a local max at $x = 0$.

$f(0) = 4$

b) $f'(x) > 0$ when $x < -1$ and when $x > 2$, $f'(x) < 0$ when $-1 < x < 2$, $f(0) = 0$



$f(x)$ is increasing when $x < -1$ and when $x > 2$.

$f(x)$ is decreasing when $-1 < x < 2$.

Therefore, there must be a local max at $x = -1$ and a local min at $x = 2$.

$f(0) = 0$

Example 4: The temperature of a person with a certain strain of flu can be approximated by the function $T(d) = -\frac{5}{18}d^2 + \frac{15}{9}d + 37$, where $0 < d < 6$; T represents the person's temperature, in degrees Celsius and d is the number of days after the person first shows symptoms. During what interval will the person's temperature be increasing?

$$T'(d) = -\frac{5}{9}d + \frac{15}{9}$$

Find critical numbers:

$$0 = -\frac{5}{9}d + \frac{15}{9}$$

$$\frac{5}{9}d = \frac{15}{9}$$

$$5d = 15$$

$d = 3$ is a critical number

0 3 6

Test value for x	1	4
$T'(d)$	$+$	$-$
$T(d)$	Increasing ↗	Decreasing ↘

Therefore, a person's temperature will be increasing over the first 3 days.