L2 – 4.4 Compound Angle Formulas
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Compound angle: *an angle that is created by adding or subtracting two or more angles.*

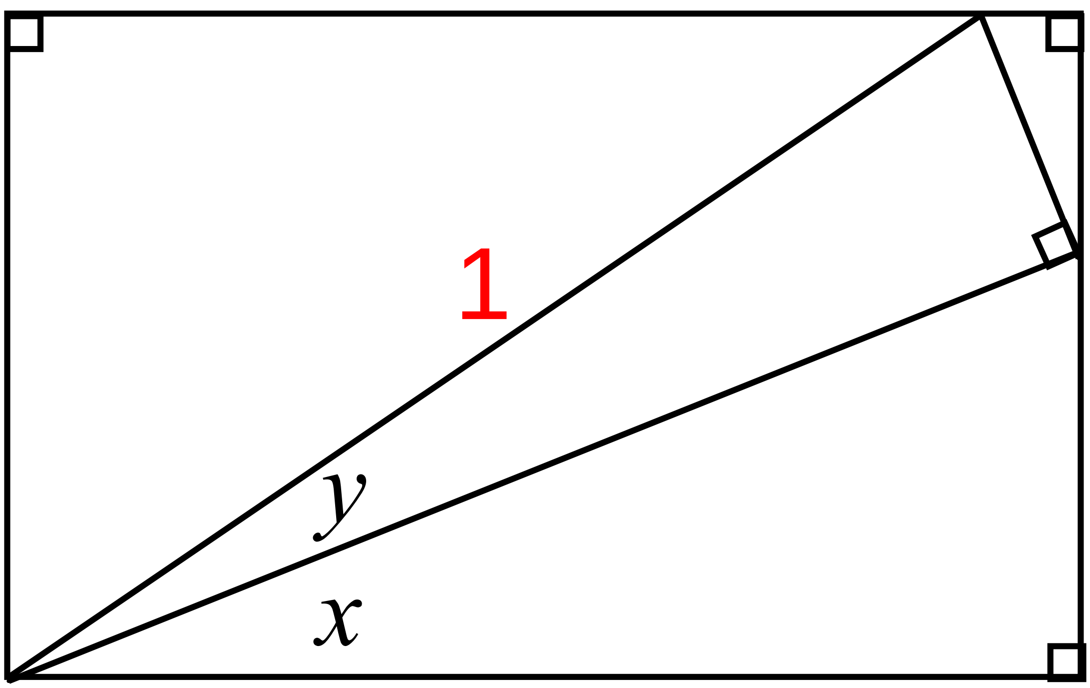
Normal algebra rules do not apply:

$$\cos(x + y) \neq \cos x + \cos y$$

Part 1: Proof of $\cos(x + y)$ and $\sin(x + y)$

So what does $\cos(x + y) =?$

Using the diagram below, label all angles and sides:



$\cos(x + y) =$

$\sin(x + y) =$

Part 2: Proofs of other compound angle formulas

Even/Odd Properties

$\cos(-x) = \cos x$ $\sin(-x) = -\sin x$

Example 1: Prove $\cos(x - y) = \cos x \cos y + \sin x \sin y$

LS	**RS**

LS = RS

Example 2:

a) Prove $\sin(x - y) = \sin x \cos y - \cos x \sin y$

LS	**RS**

LS = RS

Compound Angle Formulas

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

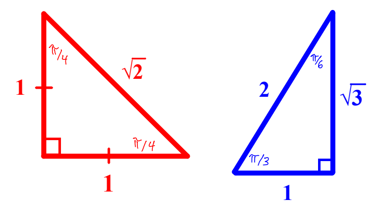
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Part 3: Determine Exact Trig Ratios for Angles other than Special Angles



By expressing an angle as a sum or difference of angles in the special triangles, exact values of other angles can be determined.

Example 3: Use compound angle formulas to determine exact values for

a) $\sin\frac{\pi}{12}$

$\sin\dfrac{\pi}{12} =$

b) $\tan\left(-\frac{5\pi}{12}\right)$

$\tan\left(-\dfrac{5\pi}{12}\right) =$

Part 4: Use Compound Angle Formulas to Simplify Trig Expressions

Example 4: Simplify the following expression

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12}+\sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

Part 5: Application

Example 5: Evaluate $\sin(a+b)$, where a and b are both angles in the second quadrant; given $\sin a=\frac{3}{5}$ and $\sin b=\frac{5}{13}$

Start by drawing both terminal arms in the second quadrant and solving for the third side.