

L2 – 1.5 Average Rates of Change

MHF4U

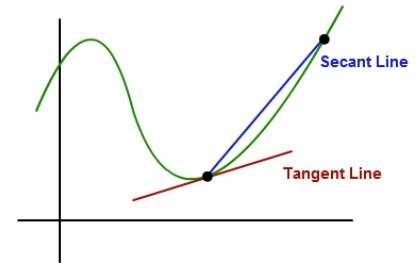
Jensen

Part 1: Terminology

Rate of Change: a measure of the change in one quantity (the dependent variable) with respect to a change in another quantity (the independent variable).

Secant Line: a line that passes through two points on the graph of a relation

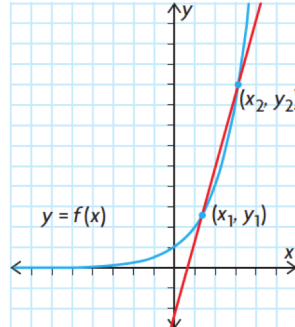
Tangent Line: a line that touches the graph of a relation at only one point within a small interval



An **average rate of change** is a change that takes place over an **interval**, while an **instantaneous rate of change** is a change that takes place in an **instant**. We will focus on average rates of change in this section.

An average rate of change corresponds to the slope of a **SECANT** between two points on a curve.

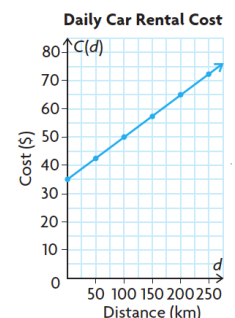
$$\text{Average rate of change} = \text{slope of secant} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$



Part 2: Average Rates of Change from a Table or Graph

Note: All **linear** relationships have a constant rate of change. Average rate of change calculations over different intervals of the independent variable give the **SAME** result.

We will be focusing on **non-linear** relationships. Non-linear relationships do not have a constant rate of change. Average rate of change calculations over different intervals of the independent interval give **DIFFERENT** results.



Example 1: Andrew drains water from a hot tub. The tub holds 1600 L of water. It takes 2 hours for the water to drain completely. The volume V , in Liters, of water remaining in the tub at various times t , in minutes, is shown in the table and graph.

a) Calculate the average rate of change in volume during each of the following time intervals.

i) $30 \leq t \leq 90$

$$m = \frac{\Delta V}{\Delta t}$$

$$= \frac{V(90) - V(30)}{90 - 30}$$

$$= \frac{100 - 900}{60}$$

$$= -13.3 \text{ L/min}$$

The volume of water is decreasing by 13.3 L/min.

ii) $60 \leq t \leq 90$

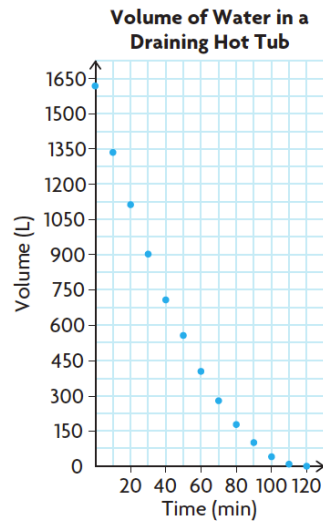
$$m = \frac{\Delta V}{\Delta t}$$

$$= \frac{V(90) - V(60)}{90 - 60}$$

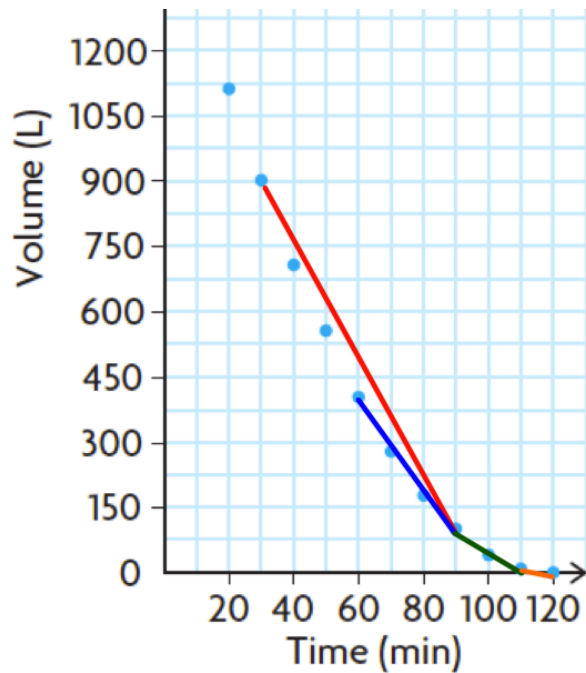
$$= \frac{100 - 400}{30}$$

$$= -10 \text{ L/min}$$

The volume of water is decreasing by 10 L/min.



Time (min)	Volume (L)
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	10
120	0



iii) $90 \leq 110$

$$\begin{aligned}m &= \frac{\Delta V}{\Delta t} \\&= \frac{V(110) - V(90)}{110 - 90} \\&= \frac{10 - 100}{20} \\&= -4.5 \text{ L/min}\end{aligned}$$

The volume of water is decreasing by 4.5 L/min.

iv) $110 \leq 120$

$$\begin{aligned}m &= \frac{\Delta V}{\Delta t} \\&= \frac{V(120) - V(110)}{120 - 110} \\&= \frac{0 - 10}{10} \\&= -1 \text{ L/min}\end{aligned}$$

The volume of water is decreasing by 1 L/min.

b) Does the tub drain at a constant rate?

The tub does not drain at a constant rate. This can be seen from the graph which shows a non-linear relationship. The rate slows as the water empties because of the pressure decrease.

A **negative** rate of change indicates the quantity of the dependent variable is decreasing over the interval. The secant line has a negative slope.

A **positive** rate of change indicates the quantity of the dependent variable is increasing over the interval. The secant line has a positive slope.

Part 2: Average Rate of Change from an Equation

Example 2: A rock is tossed upward from a cliff that is 120 meters above the water. The height of the rock above the water is modelled by $h(t) = -5t^2 + 10t + 120$, where h is the height in meters and t is the time in seconds. Calculate the average rate of change in height during each time intervals.

a) $0 \leq t \leq 1$

$$\begin{aligned} m &= \frac{\Delta h}{\Delta t} \\ &= \frac{h(1) - h(0)}{1 - 0} \\ &= \frac{125 - 120}{1} \\ &= 5 \text{ m/s} \end{aligned}$$

b) $1 \leq t \leq 2$

$$\begin{aligned} m &= \frac{\Delta h}{\Delta t} \\ &= \frac{h(2) - h(1)}{2 - 1} \\ &= \frac{120 - 125}{1} \\ &= -5 \text{ m/s} \end{aligned}$$

c) $2 \leq t \leq 3$

$$\begin{aligned} m &= \frac{\Delta h}{\Delta t} \\ &= \frac{h(3) - h(2)}{3 - 2} \\ &= \frac{105 - 120}{1} \\ &= -15 \text{ m/s} \end{aligned}$$