

<https://www.geogebra.org/3d/aku6q7xs>

In two-space lines can be represented using: vector equations, parametric equations, scalar equations, or an equation in slope  $y$ -intercept form.

A line in three space can be defined by a vector, parametric, or symmetric equation but not a scalar equation. In three-space, a scalar equation defines a plane. A plane is a two-dimensional flat surface that extends infinitely in all directions.

As in two-space, a line in three-space needs a position vector to a known point on the line and a direction vector in order to define it.

### Equations of a line in $R^3$ :

i) Vector Equation  $\rightarrow \vec{r} = \vec{r}_0 + t\vec{m}$  or  $[x, y, z] = [x_0, y_0, z_0] + t[m_1, m_2, m_3]$

ii) Parametric Equation  $\rightarrow \begin{cases} x = x_0 + tm_1 \\ y = y_0 + tm_2 \\ z = z_0 + tm_3 \end{cases}$

where  $\vec{r}_0$  is the position vector and  $\vec{m}$  is the direction vector.

iii) Symmetric Equation  $\rightarrow \frac{x-x_0}{m_1} = \frac{y-y_0}{m_2} = \frac{z-z_0}{m_3}$

The symmetric equation is derived from the parametric equations and solving for the  $t$  parameter in each component.

**Example 1:** A line passes through points  $A(2, -1, 5)$  and  $B(3, 6, -4)$ .

a) Write a vector equation of the line.

$$\vec{r}_0 = [2, -1, 5]$$

$$\vec{m} = \overrightarrow{AB}$$

$$\vec{m} = [3, 6, -4] - [2, -1, 5]$$

$$\vec{m} = [1, 7, -9]$$

The vector equation is  $[x, y, z] = [2, -1, 5] + t[1, 7, -9]$

b) Write parametric equations for the line.

$$\ell: \begin{cases} x = 2 + t \\ y = -1 + 7t \\ z = 5 - 9t \end{cases}$$

c) Determine if the point  $C(0, -15, 9)$  lies on the line.

Sub the point in to the parametric equations and see if there is a single value of  $t$  that makes the equations true:

$$0 = 2 + t$$

$$t = -2$$

$$-15 = -1 + 7t$$

$$t = -2$$

$$9 = 5 - 9t$$

$$t = \frac{-4}{9}$$

Therefore, the point is NOT on the line.

**Example 2:** Find Vector, Parametric, and Symmetric equations of a line that passes through points  $A(2, -1, 3)$  and  $B(5, 1, 1)$ .

**Vector Equation:**

$$\vec{r}_0 = [2, -1, 3]$$

$$\vec{m} = [5, 1, 1] - [2, -1, 3]$$

$$\vec{m} = [3, 2, -2]$$

Vector equation is  $[x, y, z] = [2, -1, 3] + t[3, 2, -2]$

**Parametric Equations:**

$$\ell: \begin{cases} x = 2 + 3t \\ y = -1 + 2t \\ z = 3 - 2t \end{cases}$$

**Symmetric Equations:**

$$x = 2 + 3t$$

$$t = \frac{x - 2}{3}$$

$$y = -1 + 2t$$

$$t = \frac{y + 1}{2}$$

$$z = 3 - 2t$$

$$t = \frac{z - 3}{-2}$$

$$\frac{x - 2}{3} = \frac{y + 1}{2} = \frac{z - 3}{-2}$$

*Notice that you cannot write a scalar equation of a line in 3-space. This is reserved for defining planes which we will do next lesson.*