L3 – Equations of Planes Unit 6

MCV4U

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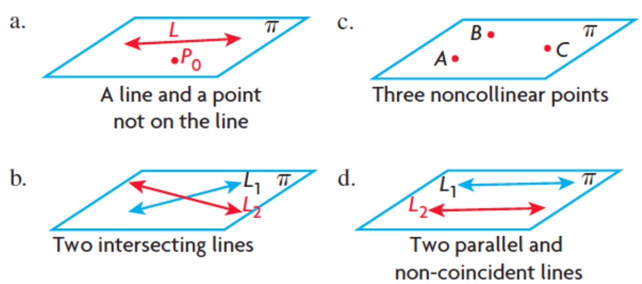
To determine the equation of a plane you need to know two non-collinear vectors parallel to the plane and a point on the plane. We can determine this information if any of the following information is given:



The equation of a plane can be written in the following forms:

i) Vector

ii) Parametric

iii) Scalar (next lesson)

Vector Equation of a Plane	**Parametric Equation of a Plane**
$\vec{r} = \vec{r_0} + t\vec{a} + s\vec{b}$ OR $[x, y, z] = [x_0, y_0, z_0] + t[a_1, a_2, a_3] + s[b_1, b_2, b_3]$ $\vec{r} = [x, y, z]$ is a position vector for any point on the plane. $\vec{r_0} = [x_0, y_0, z_0]$ is a position vector for a known point on the plane. $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$ are non-parallel direction vectors parallel to the plane. t and s are scalars	$\begin{cases} x = x_0 + ta_1 + sb_1 \\ y = y_0 + ta_2 + sb_2 \\ z = z_0 + ta_2 + sb_2 \end{cases}$

Example 1: Consider the plane with direction vectors $\vec{a} = [8, -5, 4]$ and $\vec{b} = [1, -3, -2]$ through $P_0(3,7,0)$.

a) Write the vector and parametric equations of the plane.

b) Determine if the point $Q(-10, 8, -6)$ is on the plane.

If the point is on the plane, then there exists a single set of t and s values that satisfy the equations.

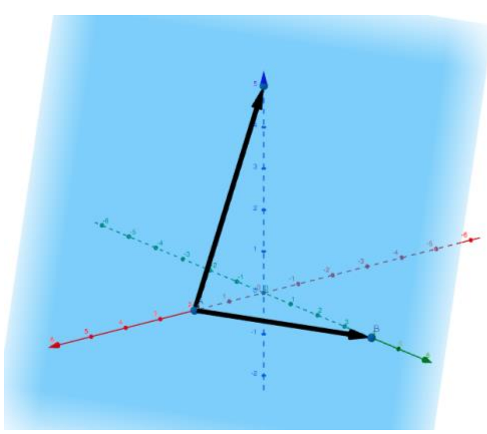
c) Find the coordinates of two other points on the plane.

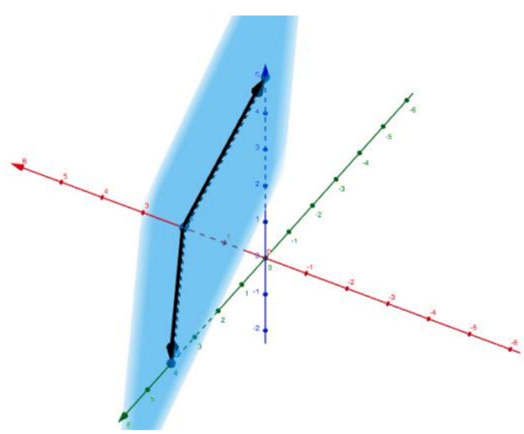
d) Find the x-intercept of the plane.

Example 2: Find the vector and parametric equations of each plane.

a) The plane with x-intercept = 2, y-intercept = 4, and z-intercept = 5.

3D visualization: https://www.geogebra.org/3d/nsj2rkzf





b) The plane containing the line $[x, y, z] = [0,3,-5] + t[6,-2,-1]$ and parallel to the line $[x, y, z] =$ $[1,7,-4] + s[1,-3,3]$.