

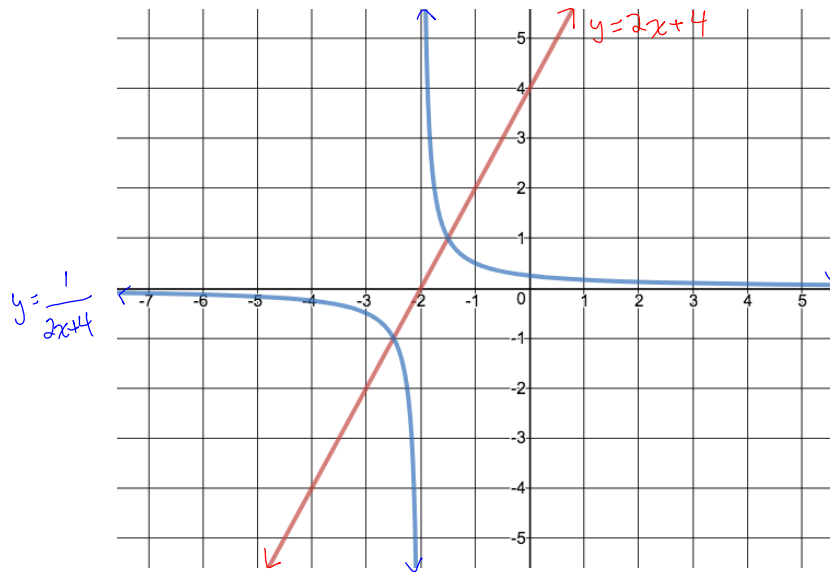
### L3 – 3.1/3.2 Reciprocal of Linear and Quadratic Functions

MHF4U

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#### Part 1: Analyze the Reciprocal of a Linear Function

Example 1:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

b) What graphical characteristic in the reciprocal function does the zero ( $x$ -int) of the original function correspond to?

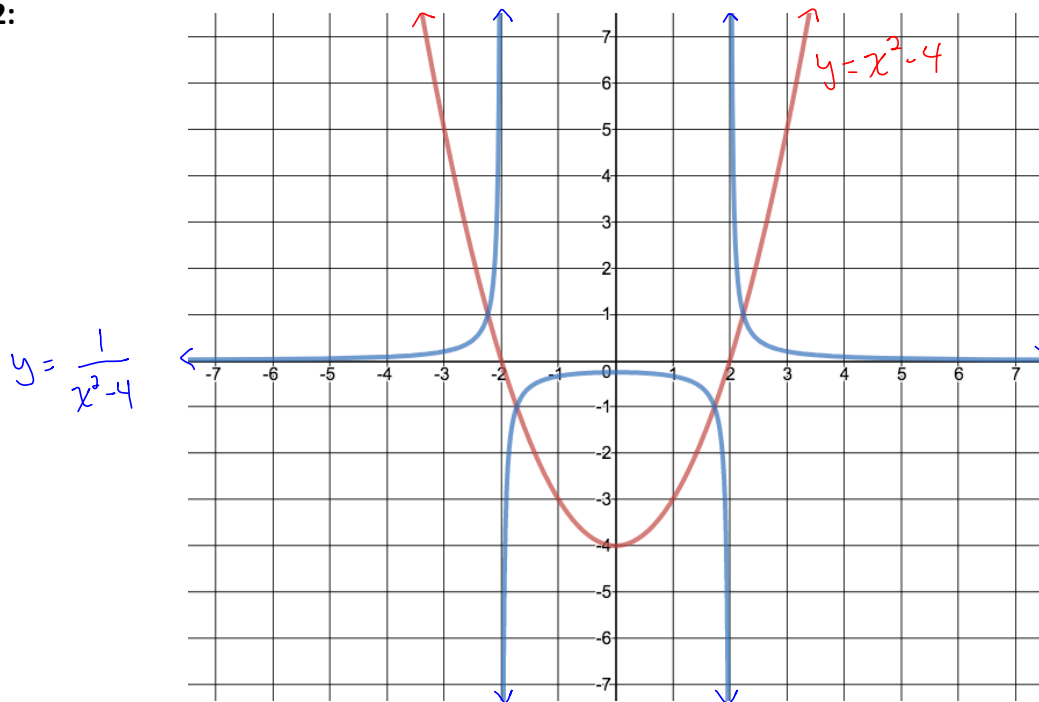
c) When the original function is increasing, what is happening to the reciprocal function?

d) What are the  $y$ -coordinates of the points of intersection?

e) Label a point on the graph of both functions at  $x = 2$ . What do you notice about the  $y$  values of each point?

## Part 2: Analyze the Reciprocal of a Quadratic Function

Example 2:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

b) What graphical characteristic in the reciprocal function do the zeros ( $x$ -int) of the original function correspond to?

c) When the original function is decreasing, what is happening to the reciprocal function?

d) What are the  $y$ -coordinates of the points of intersection?

f) Label the local min or max point on each function. What do you notice about them?

## Properties of Reciprocal Functions

- All the  $y$ -coordinates of the reciprocal function are the reciprocals of the  $y$ -coordinates of the original function
- The graph of the reciprocal function has a vertical asymptote at the  $x$ -intercepts (zeros) of the original
  - This is because it makes the denominator of the reciprocal = 0
- $y = 0$  will always be a horizontal asymptote
- The reciprocal function has the same positive/negative intervals as the original function
- Intervals of increase on the original function are intervals of decrease on the reciprocal
- Intervals of decrease on the original function are intervals of increase on the reciprocal
- If 1 is in the range of the original function, this is where the functions will intersect
- If the original function has a local min point, the reciprocal will have a local max at the same  $x$ -value (and vice versa)

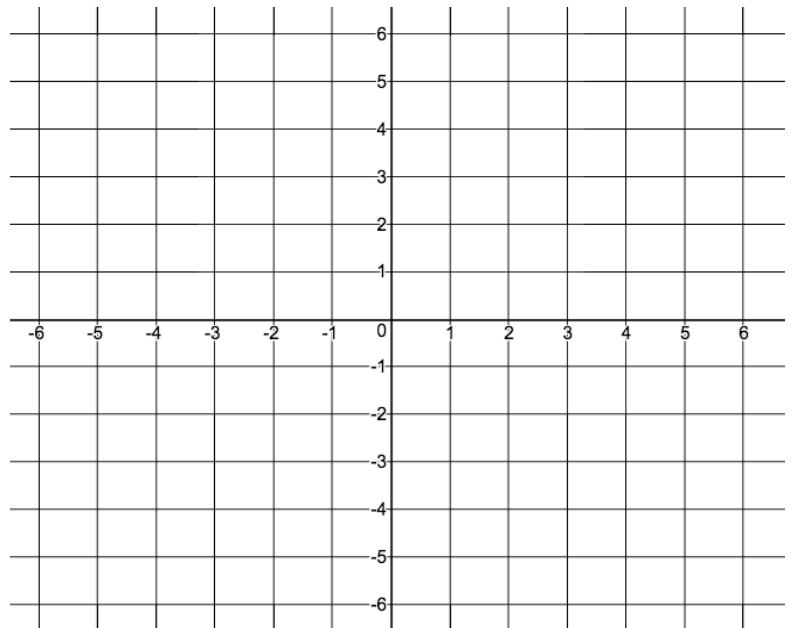
## Part 3: Graphing Reciprocal Functions

### Process:

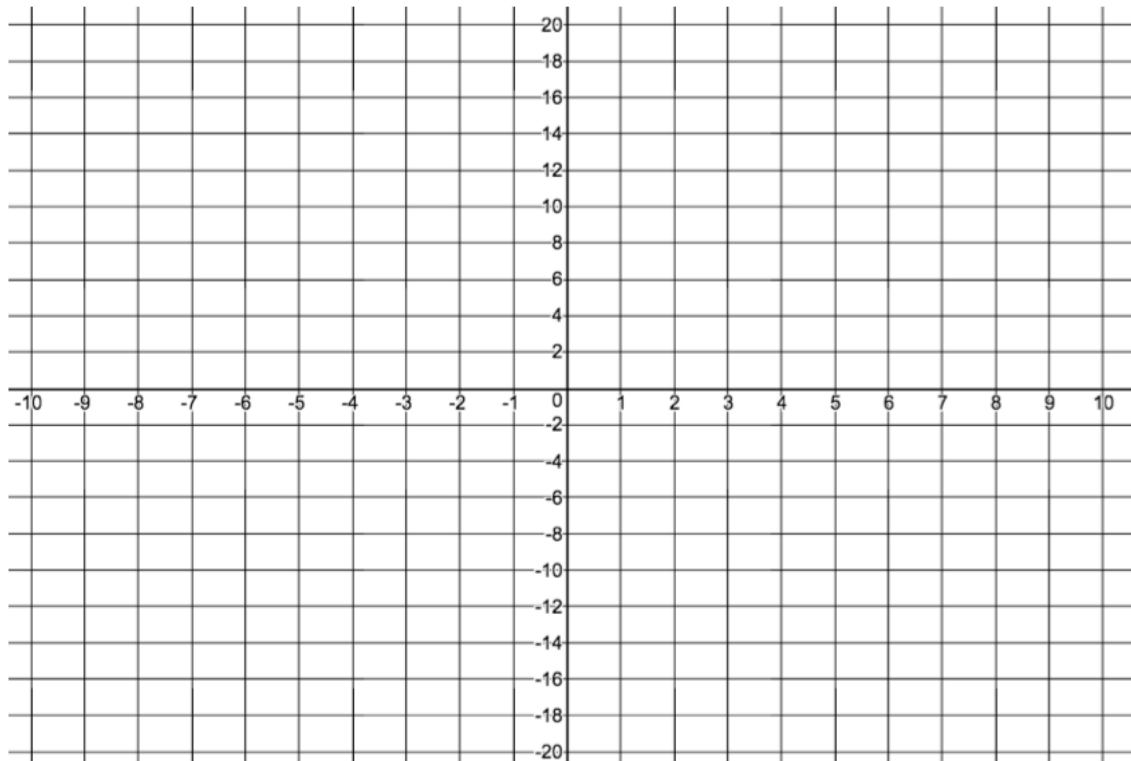
- Find key features of the function in the denominator and graph it using a table of values
- Create a table of values for the reciprocal function by keeping the same  $x$  values but using the reciprocal of all  $y$  values
- Draw vertical asymptotes at any point that is a zero of the original linear/quadratic function
  - Reciprocal of 0 is undefined
- If the numerator is something other than 1, multiply the  $y$ -values by this stretch factor

**Example 3:** Graph each of the following reciprocal functions. Start by graphing the function in the denominator.

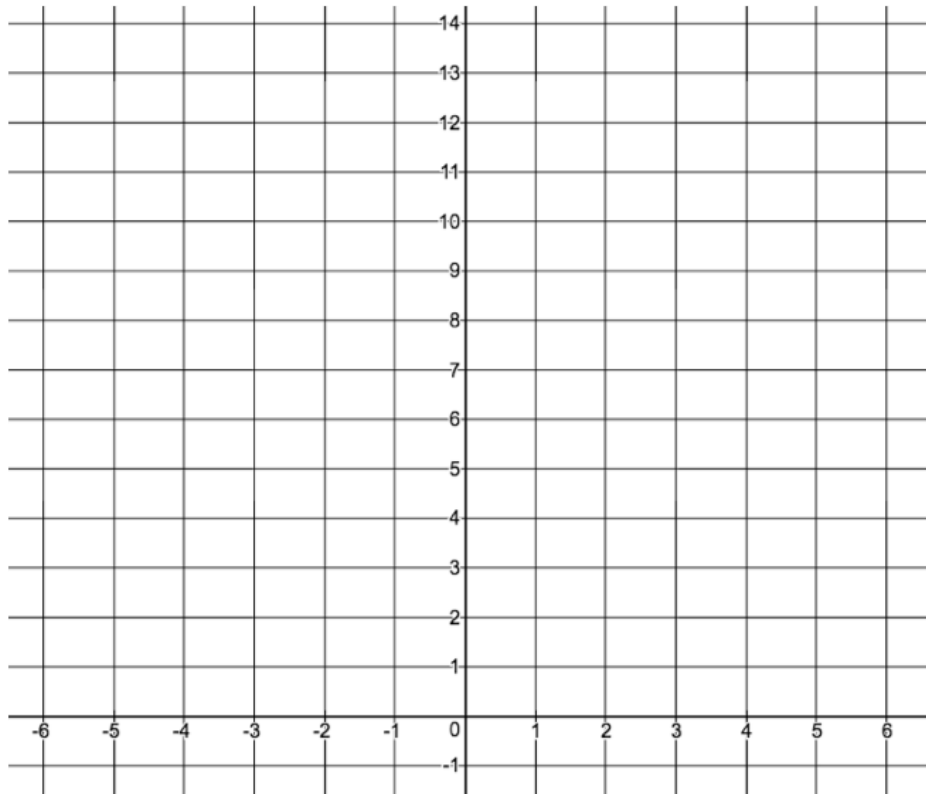
a)  $y = \frac{1}{2x-1}$



b)  $y = \frac{1}{x^2 - 2x - 15}$



c)  $y = \frac{1}{x^2+4}$



$$\mathbf{d)} \ y = \frac{2}{x^2 - 6x + 9} = 2 \left[ \frac{1}{(x-3)^2} \right]$$

