L4 – Rational Functions Unit 2

MCV4U

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Part 1: Warm-Up

Find the intervals of concavity and the coordinates of any points of inflection for $y = \frac{1}{3}x^3 - 12x^2 + 5$

Remember:

$f''(x) = 0$ or undefined is a possible POI

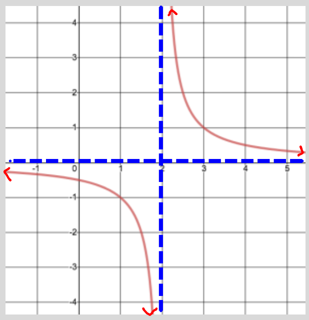
If $f''(x) < 0$, $f(x)$ is concave DOWN

If $f''(x) > 0$, $f(x)$ is concave UP

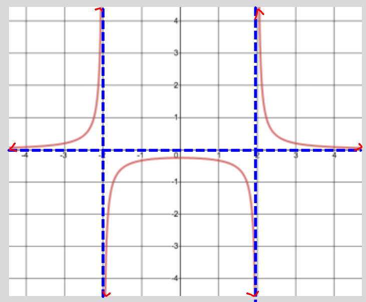
<u>Part 2: Reminder of some simple rational functions</u>

Degree of denominator > degree of numerator:

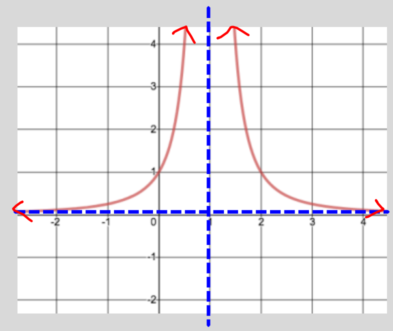
$y = \frac{1}{x-2}$



$y = \frac{1}{x^2-4}$



$y = \frac{1}{(x-1)^2}$

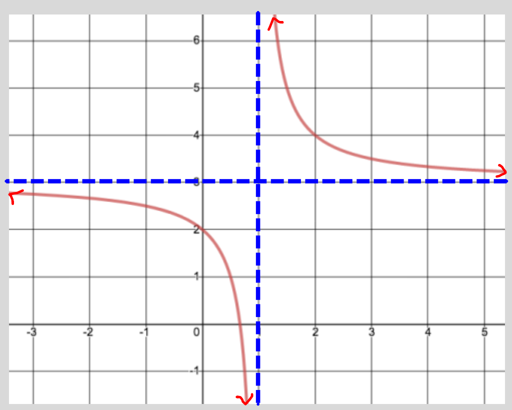


Notice: Horizontal asymptotes all are at $y = 0$

Vertical asymptotes are at zeros of the denominator

Degree of denominator = degree of numerator:

$y = \frac{3x-2}{x-1}$

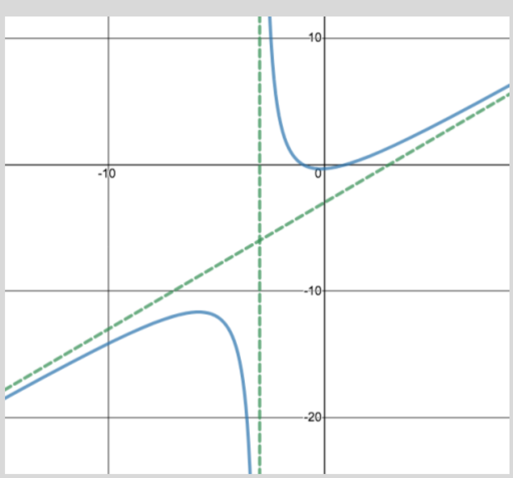


Notice: HA at quotient of leading coefficients

VA at zero of the denominator

Degree of denominator < degree of numerator:

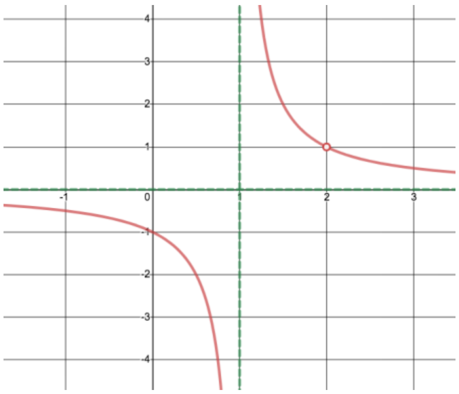
$y = \frac{x^2-1}{x+3}$



Notice: Oblique asymptote at quotient of numerator and denominator; VA at zero of the denominator

Vertical Asymptote vs. Hole in Graph

$f(x) = \frac{(x-2)}{(x-1)(x-2)}$



Notice: VA at $x = 1$; $f(1) = \frac{-1}{0}$

Hole at $(2, 1)$; $f(2) = \frac{0}{0}$

(remove discontinuity to find y-value of hole)

Conclusion: If $f(a) = \frac{\#}{0}$, $x = a$ is a VA

If $f(a) = \frac{0}{0}$, there is a hole in the graph when $x = a$

Limit Definition of Asymptotes:

For the rational function $y = \frac{f(x)}{g(x)}$

There is a Vertical Asymptote at $x = a$ when $g(a) = 0$ and $\lim_{x \to a} \frac{f(x)}{g(x)} = \pm\infty$

There is a Horizontal Asymptote at $y = L$ when $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = L$

Note: Horizontal asymptote only exists if the degree of the numerator is ________________________________ the degree of the denominator.

__**Part 3: Apply What You Know to Graph Rational Functions**__

Example 1: State the Horizontal Asymptotes of the following functions:

a) $y = \frac{3x^2+2}{6x^2-4x-1}$

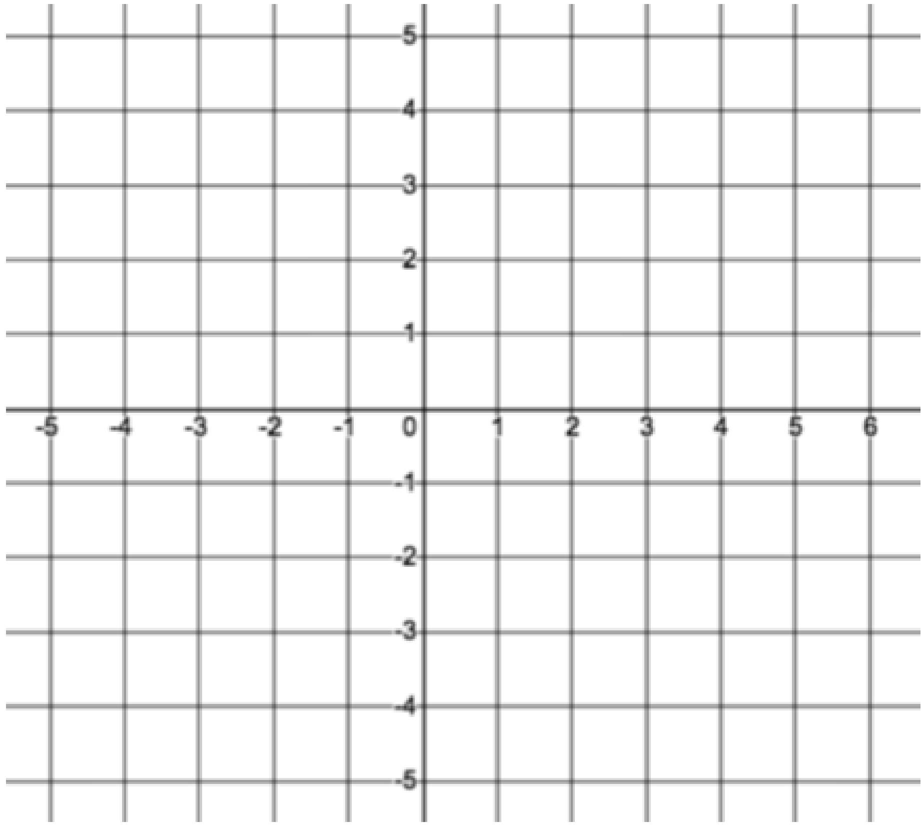
b) $y = \frac{3x^2+2}{6x^3-4x-1}$

Example 2: Consider the function $f(x) = \frac{1}{(x+2)(x-3)}$

a) Find the asymptotes

b) Find the one-sided limits as the x-values approach the vertical asymptotes (sub values very close to the limit for x, and find what the value of the function is approaching)

c) Sketch the graph



Example 3: Consider the function $f(x) = \frac{1}{x^2+1}$

a) Where are the vertical and horizontal asymptotes?

b) Find any local max/min points and the intervals of increase/decrease

c) Find the points of inflection

d) Sketch a graph of the function