

## L4 – 4.5 Prove Trig Identities

MHF4U

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Using your sheet of all identities learned this unit, prove each of the following:

**Example 1:** Prove  $\frac{\sin(2x)}{1+\cos(2x)} = \tan x$

<u>LS</u>		<u>RS</u>
$= \frac{\sin(2x)}{1+\cos(2x)}$		$= \tan x$
$= \frac{\sin(2x)}{1+2\cos^2 x - 1}$		$= \frac{\sin x}{\cos x}$
$= \frac{\cancel{2} \sin x \cancel{\cos x}}{\cancel{2} \cos^2 x}$		
$= \frac{\sin x}{\cos x}$		
	LS = RS	

**Example 2:** Prove  $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

<u>LS</u>		<u>RS</u>
$= \cos\left(\frac{\pi}{2} + x\right)$		$= -\sin x$
$= \cos\left(\frac{\pi}{2}\right)\cos x - \sin\left(\frac{\pi}{2}\right)\sin x$		
$= 0(\cos x) - 1\sin x$		
$= -\sin x$		
	LS = RS	

**Example 3:** Prove  $\csc(2x) = \frac{\csc x}{2 \cos x}$

LS

$$\begin{aligned} &= \csc(2x) \\ &= \frac{1}{\sin(2x)} \\ &= \frac{1}{2 \sin x \cos x} \end{aligned}$$

RS

$$\begin{aligned} &= \frac{\csc x}{2 \cos x} \\ &= \csc x \cdot \frac{1}{2 \cos x} \\ &= \frac{1}{\sin x} \cdot \frac{1}{2 \cos x} \\ &= \frac{1}{2 \sin x \cos x} \end{aligned}$$

LS=RS

**Example 4:** Prove  $\cos x = \frac{1}{\cos x} - \sin x \tan x$

LS

$$= \cos x$$

RS

$$\begin{aligned} &= \frac{1}{\cos x} - \sin x \tan x \quad \text{QI} \\ &= \frac{1}{\cos x} - \sin x \left( \frac{\sin x}{\cos x} \right) \\ &= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} \\ &= \frac{\cos^2 x}{\cos x} \quad \text{PI} \\ &= \cos x \end{aligned}$$

LS=RS

**Example 5:** Prove  $\tan(2x) - 2 \tan(2x) \sin^2 x = \sin 2x$

<u>LS</u>		<u>RS</u>
$= \tan(2x) - 2 \tan(2x) \sin^2 x$ $= \tan(2x) [1 - 2 \sin^2 x]$ $= \tan(2x) \cos(2x)$ $= \frac{\sin(2x)}{\cos(2x)} \cdot \cos(2x)$ $= \sin(2x)$	$= \sin(2x)$	
	$LS = RS$	

**Example 6:** Prove  $\frac{\cos(x-y)}{\cos(x+y)} = \frac{1 + \tan x \tan y}{1 - \tan x \tan y}$

<u>LS</u>		<u>RS</u>
$= \frac{\cos(x-y)}{\cos(x+y)}$ $= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$	$= \frac{1 + \tan x \tan y}{1 - \tan x \tan y}$ $= \frac{1 + \left(\frac{\sin x}{\cos x}\right) \left(\frac{\sin y}{\cos y}\right)}{1 - \left(\frac{\sin x}{\cos x}\right) \left(\frac{\sin y}{\cos y}\right)} \times \frac{\cos x \cos y}{\cos x \cos y}$ $= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$	