

## L4 – 5.3 Transformations of Trig Functions

MHF4U

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### Part 1: Transformation Properties

$$y = a \sin[k(x - d)] + c$$

[Desmos Demonstration](#)

$a$	$k$	$d$	$c$
Vertical stretch or compression by a factor of $a$ .	Horizontal stretch or compression by a factor of $\frac{1}{ k }$ .	Phase shift $d > 0$ ; shift right	Vertical shift $c > 0$ ; shift up
Vertical reflection if $a < 0$ $ a  = \text{amplitude}$	Horizontal reflection if $k < 0$ . $\frac{2\pi}{ k } = \text{period}$	$d < 0$ ; shift left	$c < 0$ ; shift down

**Example 1:** For the function  $y = 3 \sin\left[\frac{1}{2}\left(\theta + \frac{\pi}{3}\right)\right] - 1$ , state the...

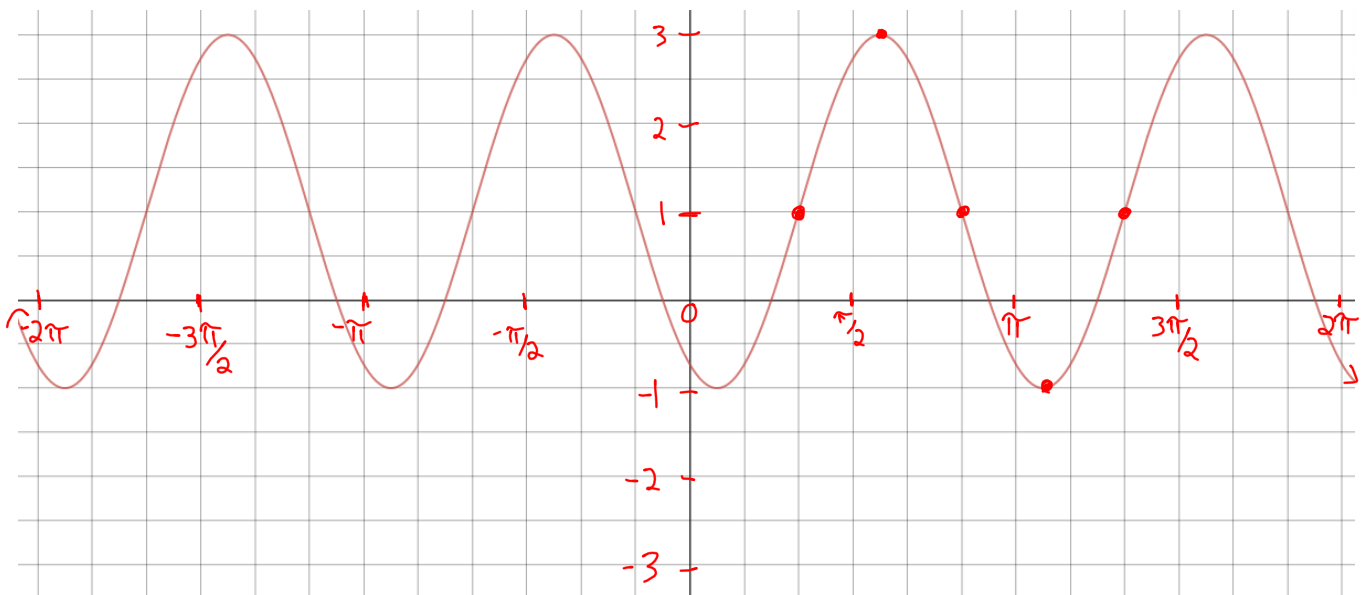
Amplitude: $amp =  a  = 3$	Period: $period = \frac{2\pi}{ k } = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$
Phase shift: $d = -\frac{\pi}{3}$ ; shift left $\frac{\pi}{3}$	Vertical shift: $c = -1$ ; shift down 1
Max: $max = c + amp = -1 + 3 = 2$	Min: $min = c - amp = -1 - 3 = -4$

## Part 2: Given Equation → Graph Function

**Example 2:** Graph  $y = 2 \sin \left[ 2 \left( x - \frac{\pi}{3} \right) \right] + 1$  using transformations. Then state the amplitude and period of the function.

$y = \sin x$	
$x$	$y$
0	0
$\frac{\pi}{2}$	1
$\pi$	0
$\frac{3\pi}{2}$	-1
$2\pi$	0

$y = 2 \sin \left[ 2 \left( x - \frac{\pi}{3} \right) \right] + 1$	
$\frac{x}{2} + \frac{\pi}{3}$	$2y + 1$
$\frac{\pi}{3} = \frac{2\pi}{6}$	1
$\frac{7\pi}{12} = \frac{3.5\pi}{6}$	3
$\frac{5\pi}{6}$	1
$\frac{13\pi}{12} = \frac{6.5\pi}{6}$	-1
$\frac{4\pi}{3} = \frac{8\pi}{6}$	1



Amplitude:  $\frac{\max - \min}{2} = \frac{3 - (-1)}{2} = 2$

Period:  $\pi$  radians

**Part 3: Given the Graph → Write the Equation**

$$y = a \sin[k(x - d)] + c$$

$a$	$k$	$d$	$c$
Find the amplitude of the function:  $a = \frac{\text{max} - \text{min}}{2}$	Find the period (in radians) of the function using a starting point and ending point of a full cycle.  $k = \frac{2\pi}{\text{period}}$	<b>for sin x:</b> trace along the center line and find the distance between the y-axis and the bottom left of the closest rising midline.  <b>for cos x:</b> the distance between the y-axis and the closest maximum point  $d_{\sin} = d_{\cos} - \frac{\pi}{2k}$  $d_{\cos} = d_{\sin} + \frac{\pi}{2k}$	Find the vertical shift  $c = \text{max} - \text{amplitude}$ OR $c = \frac{\text{max} + \text{min}}{2}$  (this finds the 'middle' of the function)

**Example 3:** Determine the equation of a sine and cosine function that describes the following graph

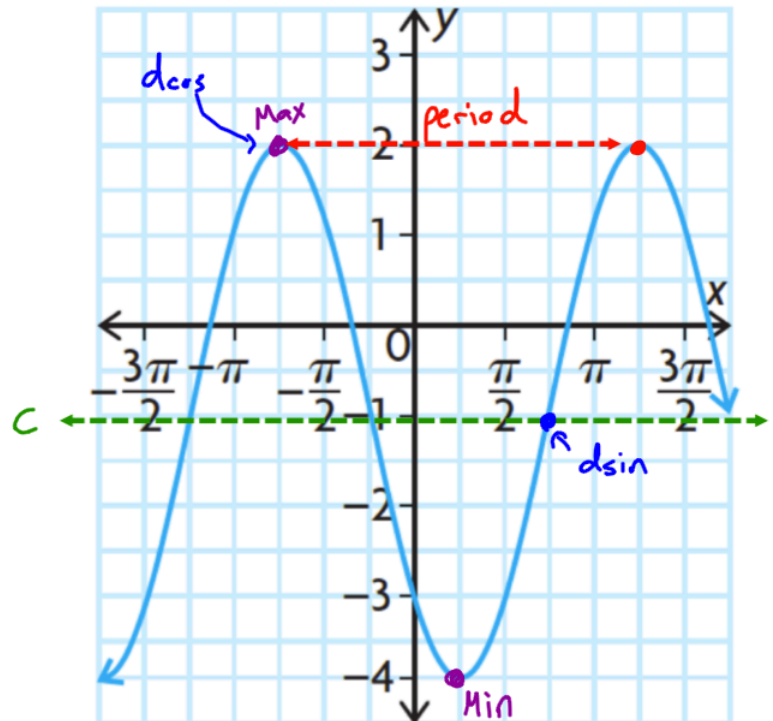
$$a = \frac{\text{max} - \text{min}}{2} = \frac{2 - (-4)}{2} = 3$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{\left(\frac{5\pi}{4} - \left(-\frac{3\pi}{4}\right)\right)} = \frac{2\pi}{2\pi} = 1$$

$$c = \text{max} - |a| = 2 - 3 = -1$$

$$d_{\cos} = -\frac{3\pi}{4}$$

$$d_{\sin} = \frac{3\pi}{4}$$



$$y = 3 \sin\left(x - \frac{3\pi}{4}\right) - 1$$

$$y = 3 \cos\left(x + \frac{3\pi}{4}\right) - 1$$

**Example 4:** Determine the equation of a sine and cosine function that describes the following graph

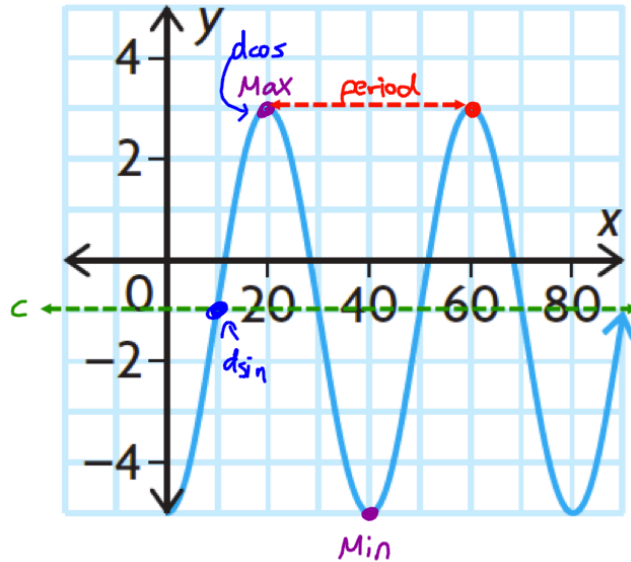
$$a = \frac{\text{max} - \text{min}}{2} = \frac{3 - (-5)}{2} = 4$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{40} = \frac{\pi}{20}$$

$$c = \text{max} - |a| = 3 - 4 = -1$$

$$d_{\cos} = 20$$

$$d_{\sin} = 10$$



$$y = 4 \cos \left[ \frac{\pi}{20} (x - 20) \right] - 1$$

$$y = 4 \sin \left[ \frac{\pi}{20} (x - 10) \right] - 1$$

**Example 5:**

a) Create a sine function with an amplitude of 7, a period of  $\pi$ , a phase shift of  $\frac{\pi}{4}$  right, and a vertical displacement of -3.

$$a = 7$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$

$$c = -3$$

$$d = \frac{\pi}{4}$$

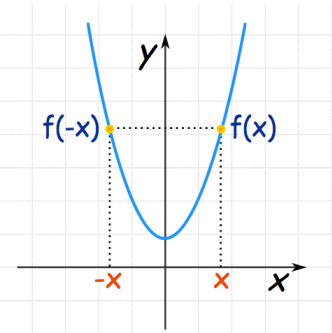
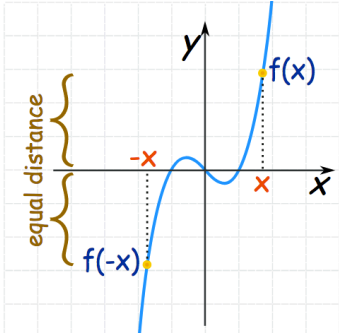
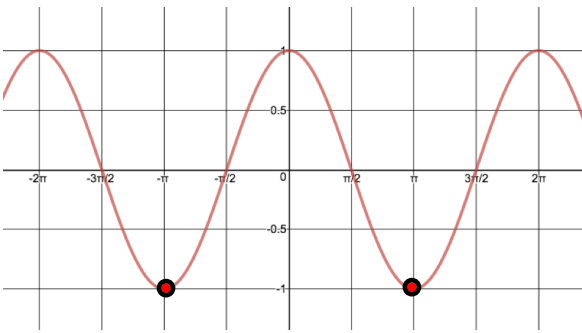
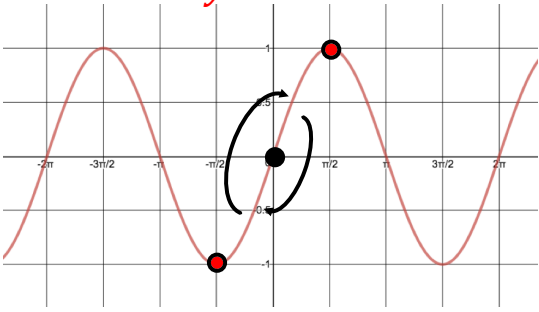
$$y = 7 \sin \left[ 2 \left( x - \frac{\pi}{4} \right) \right] - 3$$

b) What would be the equation of a cosine function that represents the same graph as the sine function above?

$$d_{\cos} = d_{\sin} + \frac{\pi}{2k} = \frac{\pi}{4} + \frac{\pi}{2(2)} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$y = 7 \cos \left[ 2 \left( x - \frac{\pi}{2} \right) \right] - 3$$

## Part 4: Even and Odd Functions

Even Functions	Odd Functions
<p>EVEN FUNCTION if:</p> <p>Line symmetry over the <u>y-axis</u></p>	<p>ODD FUNCTION if:</p> <p>Point symmetry about the <u>origin (0, 0)</u></p>
<p>Rule:</p> $f(-x) = f(x)$ 	<p>Rule:</p> $-f(x) = f(-x)$ 
<p>Example:</p> $y = \cos x$  $f(\pi) = -1$ $f(-\pi) = -1$ <p>Therefore,</p> $f(\pi) = f(-\pi)$	<p>Example:</p> $y = \sin x$  $f\left(\frac{\pi}{2}\right) = 1$ $f\left(-\frac{\pi}{2}\right) = -1$ <p>Therefore,</p> $-f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$ <p><math>y = \tan x</math> is also an odd function</p> 