L4 – Equations of Planes in Scalar Form Unit 6
MCV4U
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The scalar equation of a plane in three-space is $Ax + By + Cz + D = 0$, where $\vec{n} = [A, B, C]$ is a normal vector to the plane.

Example 1a: Determine the scalar equation of a plane that has normal vector $\vec{n} = [3, -2, 5]$ and contains the point $P_0(1, 2, -3)$.

Method 1: Dot Product	**Method 2: Use $Ax + By + Cz + D = 0$**
Let $P(x, y, z)$ be any point on the plane $\overrightarrow{PP_0} \cdot \vec{n} = 0$ $[1 - x, 2 - y, -3 - z] \cdot [3, -2, 5] = 0$ $3 - 3x - 4 + 2y - 15 - 5z = 0$ $0 = 3x - 2y + 5z + 16$	The components of the normal vector are the coefficients of the scalar equation: $3x - 2y + 5z + D = 0$ Solve for the unknown constant by subbing in the known point on the plane: $3(1) - 2(2) + 5(-3) + D = 0$ $D = 16$ The equation is $3x - 2y + 5z + 16 = 0$

Example 1b: Is vector $\vec{a} = [4, 1, -2]$ parallel to the plane?

$\vec{a}$ is parallel to the plane if $\vec{a} \cdot \vec{n} = 0$

$\vec{a} \cdot \vec{n} = [4, 1, -2] \cdot [3, -2, 5]$

$\vec{a} \cdot \vec{n} = 4(3) + 1(-2) + (-2)(5)$

$\vec{a} \cdot \vec{n} = 0$

Therefore $\vec{a}$ is parallel to the plane.

Example 1c: Is vector $\vec{b} = [15, -10, 25]$ normal to the plane?

$\vec{b}$ is perpendicular the plane if it is a scalar multiple of $\vec{n}$

$\vec{b} = k\vec{n}$

$[15, -10, 25] = k[3, -2, 5]$

$15 = 3k$	$-10 = -2k$	$25 = 5k$
$k = 5$	$k = 5$	$k = 5$

Therefore, $\vec{b}$ is normal to the plane.

Example 2: Find the scalar equation of the plane containing the points $A(-3, -1, -2)$, $B(4,6,2)$, and $C(5, -4, 1)$.

Find two direction vectors:

$\overrightarrow{AC} = [5, -4, 1] - [-3, -1, -2]$

$\overrightarrow{AC} = [8, -3, 3]$

$\overrightarrow{AB} = [4,6,2] - [-3, -1, -2]$

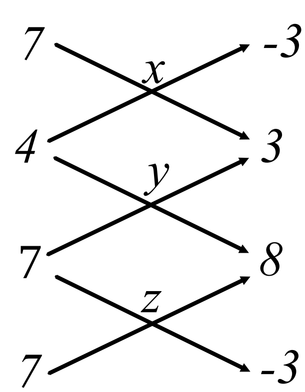
$\overrightarrow{AB} = [7,7,4]$

$\overrightarrow{AB} \times \overrightarrow{AC}$ will produce a vector perpendicular to the plane:

$\overrightarrow{AB} \times \overrightarrow{AC} = [7(3) - 4(-3), 4(8) - 7(3), 7(-3) - 7(8)]$

$\overrightarrow{AB} \times \overrightarrow{AC} = [33, 11, -77]$ (reduce)

$\vec{n} = [3, 1, -7]$



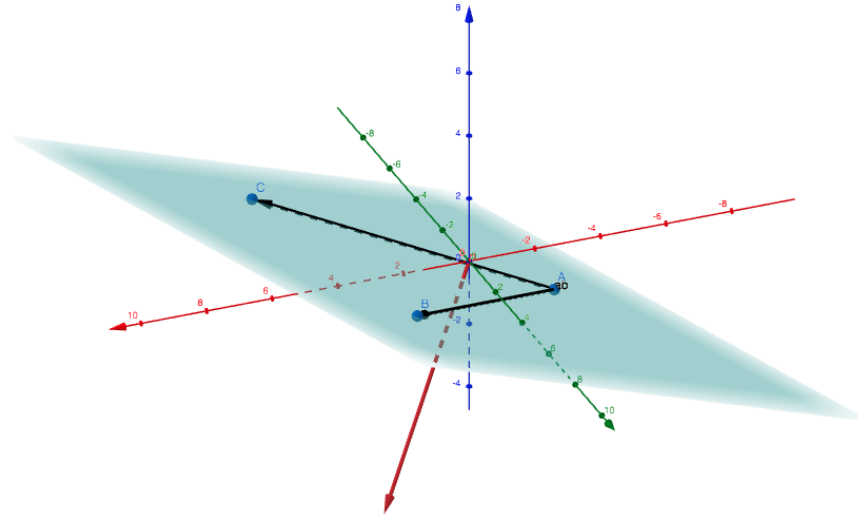
Standard form equation:

$3x + y - 7z + D = 0$

$3(4) + 1(6) - 7(2) + D = 0$

$D = -4$

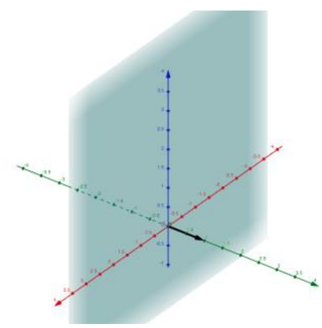
$3x + y - 7z - 4 = 0$



https://www.geogebra.org/3d/u3n69tvc

Example 3: Determine the scalar equation of each plane

a) parallel to the xz-plane; through the point (-7,8,9)



A vector parallel to the y-axis will be perpendicular to the xz-plane. Therefore, $\vec{n} = [0,1,0]$ is a possible normal vector. (see diagram)

$0x + 1y + 0z + D = 0$

$0(-7) + 1(8) + 0(9) + D = 0$

$D = -8$

$y - 8 = 0$ OR $y = 8$

Geogebra 3D visualization

b) containing the line $[x, y, z] = [1,2,4] + t[4,1,11]$ and perpendicular to $[x, y, z] = [4,15,8] + s[2,3,-1]$.

$\vec{n} = [2,3,-1]$

Point $(1,2,4)$

$2x + 3y - z + D = 0$

$2(1) + 3(2) - 4 + D = 0$

$D = -4$

$2x + 3y - z - 4 = 0$