L4 – The Quotient Rule Unit 1
MCV4U
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The Quotient Rule:

If $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

Proof:

$$h(x) = \frac{f(x)}{g(x)}$$

$$h(x)g(x) = f(x)$$

$$h'(x)g(x) + g'(x)h(x) = f'(x)$$

$$h'(x) = \frac{f'(x) - g'(x)h(x)}{g(x)}$$

$$h'(x) = \frac{f'(x) - g'(x)\frac{f(x)}{g(x)}}{g(x)}$$

$$h'(x) = \left[\frac{f'(x) - g'(x)\frac{f(x)}{g(x)}}{g(x)}\right]\left[\frac{g(x)}{g(x)}\right]$$

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

Example 1: Find the derivative of each of the following:

a) $f(x) = \frac{3x-4}{x^2+5}$

$$f'(x) = \frac{3(x^2+5) - 2x(3x-4)}{(x^2+5)^2}$$

$$f'(x) = \frac{3x^2+15-6x^2+8x}{(x^2+5)^2}$$

$$f'(x) = \frac{-3x^2+8x+15}{(x^2+5)^2}$$

b) $g(x) = \frac{6x-5}{x^3+4}$

$$g'(x) = \frac{6(x^3+4) - 3x^2(6x-5)}{(x^3+4)^2}$$

$$g'(x) = \frac{6x^3+24-18x^3+15x^2}{(x^3+4)^2}$$

$$g'(x) = \frac{-12x^3+15x^2+24}{(x^3+4)^2}$$

$$g'(x) = \frac{-3(4x^3-5x^2-8)}{(x^3+4)^2}$$

c) $h(x) = \frac{2x+8}{\sqrt{x}}$

$$h'(x) = \frac{2(\sqrt{x}) - \frac{1}{2}x^{-\frac{1}{2}}(2x+8)}{(\sqrt{x})^2}$$

$$h'(x) = \frac{2\sqrt{x} - \frac{1}{2\sqrt{x}}(2)(x+4)}{x}$$

$$h'(x) = \frac{2\sqrt{x} - \frac{1}{\sqrt{x}}(x+4)}{x} \times \frac{\sqrt{x}}{\sqrt{x}}$$

$$h'(x) = \frac{2x-1(x+4)}{x\sqrt{x}}$$

$$h'(x) = \frac{x-4}{x^{\frac{3}{2}}}$$

d) $r(x) = \frac{x+3}{\sqrt{x^2-1}}$

$$r'(x) = \frac{1(x^2-1)^{\frac{1}{2}} - \frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x)(x+3)}{\left(\sqrt{x^2-1}\right)^2}$$

$$r'(x) = \frac{(x^2-1)^{-\frac{1}{2}}\left[1(x^2-1)^1 - \frac{1}{2}(2x)(x+3)\right]}{x^2-1}$$

$$r'(x) = \frac{[1(x^2-1) - 1(x)(x+3)]}{(x^2-1)^{\frac{1}{2}}(x^2-1)^1}$$

$$r'(x) = \frac{x^2-1-x^2-3x}{(x^2-1)^{\frac{3}{2}}}$$

$$r'(x) = \frac{-3x-1}{(x^2-1)^{\frac{3}{2}}}$$

Note: *$\frac{d}{dx}\sqrt{x^2-1}$ from part d) uses the 'chain rule'. We will learn this in depth in the next lesson.*

Example 2: Determine an equation for the tangent to the curve $y = \frac{x^2-3}{5-x}$ at $x = 2$.

Point on Tangent Line:

$$y = \frac{2^2 - 3}{5 - 2}$$

$$y = \frac{1}{3}$$

$$\left(2, \frac{1}{3}\right)$$

Slope of tangent line:

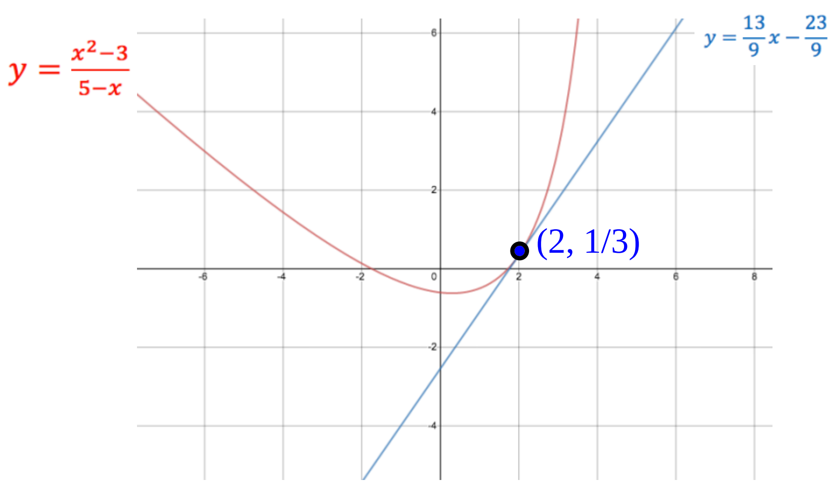
$$\frac{dy}{dx} = \frac{(2x)(5-x) - (-1)(x^2-3)}{(5-x)^2}$$

$$\frac{dy}{dx} = \frac{-x^2 + 10x - 3}{(5-x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{-(2)^2 + 10(2) - 3}{(5-2)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{13}{9}$$

Equation of tangent line:



$$y = mx + b$$

$$\frac{1}{3} = \frac{13}{9}(2) + b$$

$$\frac{3}{9} - \frac{26}{9} = b$$

$$b = -\frac{23}{9}$$

$$y = \frac{13}{9}x - \frac{23}{9}$$

Example 3: Determine the coordinates of each point on the graph of $h(x) = \frac{2x+8}{\sqrt{x}}$ where the tangent is horizontal.

The tangent will be horizontal when $h'(x) = 0$

$h'(x) = \frac{x-4}{x^{\frac{3}{2}}}$

$0 = \frac{x-4}{x^{\frac{3}{2}}}$

$0 = x - 4$

$x = 4$

$$h(4) = \frac{2(4) + 8}{\sqrt{4}}$$

$h(4) = 8$

There will be a horizontal tangent to the point (4, 8) on the graph of $h(x)$.