L6 – Intersection of Lines and Planes | Unit 6
MCV4U
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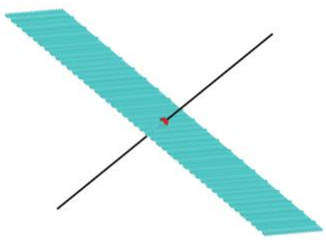
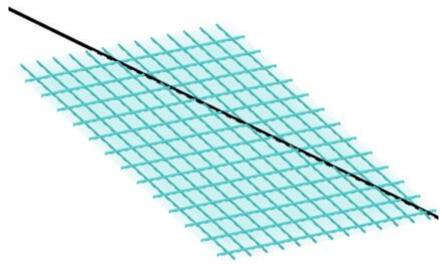
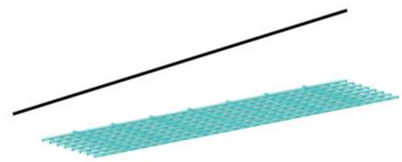
Part 1: Intersections of Lines and Planes

Given a line and plane in 3-space, there are three possibilities for the intersection of the line with the plane:

Intersect at 1 Point	Infinite Number of Solutions	No Solutions
	 The line lies on the plane	 The line is parallel and distinct from the plane

Example 1: In each case, determine if the line and the plane intersect. If so, determine the solution.

a) $\pi_1: 9x + 13y - 2z = 29$

$$\ell_1: \begin{cases} x = 5 + 2t \\ y = -5 - 5t \\ z = 2 + 3t \end{cases}$$

https://www.geogebra.org/3d/a9j3u5u2

b) $\pi_2: x + 3y - 4z = 10$

$$\ell_2: \begin{cases} x = 4 + 6t \\ y = -7 + 2t \\ z = 1 + 3t \end{cases}$$

https://www.geogebra.org/3d/quagawhg

c) $\pi_3: 4x - y + 11z = -1$

$\ell_3: [x, y, z] = [-2,4,1] + t[3,1,-1]$

https://www.geogebra.org/3d/emxyfbbx

Example 2: Without solving, determine if each line intersects the plane.

a) $\pi_1: 3x - y + z = -6$

$\ell_1: [x, y, z] = [2, -5,3] + s[3,2,1]$

b) $\pi_2: 4x - 2z = 11$

$\ell_2: [x, y, z] = [1,0,1] + s[-2,1,-4]$

Part 2: Distance from a Point to a Plane

Remember the formula for the magnitude of the projection of $\vec{a}$ on to $\vec{b}$ is $\left|proj_{\vec{b}}\ \vec{a}\right| = \frac{|\vec{a}\cdot\vec{b}|}{|\vec{b}|}$

The shortest distance between a point and a plane is the perpendicular distance. This distance, d, between a point P and a plane is $d = \left|proj_{\vec{n}}\ \overrightarrow{PQ}\right| = \frac{|\overrightarrow{PQ}\cdot\vec{n}|}{|\vec{n}|}$ where Q is ANY point on the plane and $\vec{n}$ is a normal vector to the plane.

Example 3: Find the distance between the plan $4x + 2y + z - 16 = 0$ and the point $P(10,3,-8)$.

https://www.geogebra.org/3d/r5qebzz4