L7 – Intersection of Planes

Unit 6

MCV4U

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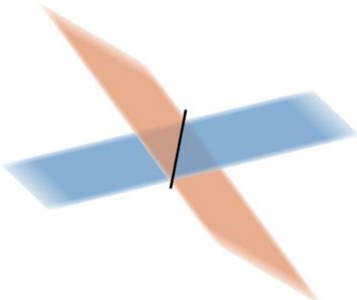
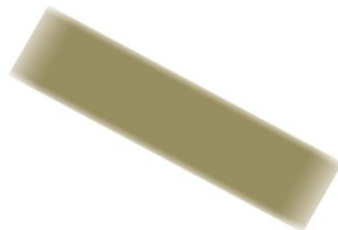
Part 1: Intersection of 2 Planes

Line of Intersection	Infinite Solutions	No Solutions
 If 2 distinct planes intersect, the solution is the set of points that lie on the line of intersection.	 If the planes are coincident, every point on the plane is a solution	 Parallel and distinct planes do not intersect

When solving a system of 2 planes, check if the planes are parallel by analyzing their normal vectors. If they are parallel, determine if they are coincident (infinite solutions) or distinct (no solutions). If the normals are not parallel, that means the planes are not parallel either. You can find the line of intersection by:

- **i)** Eliminating a variable using the method of elimination
- **ii)** Choose 1 of the remaining variables to be the parameter t
- **iii)** Write the other 2 variables in terms of the parameter t as well to get the parametric equation of the line of intersection

Example 1: Describe how the planes in each pair intersect.

a) $\pi_1: 2x - y + z - 1 = 0$

$\pi_2: x + y + z - 6 = 0$

https://www.geogebra.org/3d/jbesfzd5

b) $\pi_3: 2x - 6y + 4z - 7 = 0$

$\pi_4: 3x - 9y + 6z - 2 = 0$

https://www.geogebra.org/3d/pgsp8f68

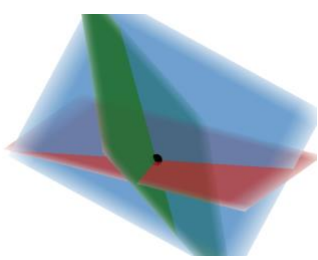
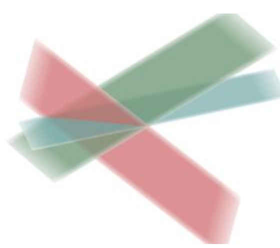
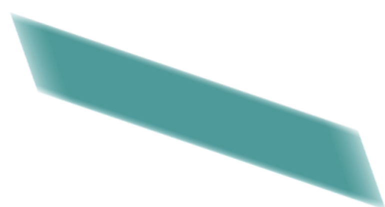
c) $\pi_5: x + y - 2z + 2 = 0$

$\pi_6: 2x + 2y - 4z + 4 = 0$

<u>Part 2: Intersection of 3 Planes</u>

A system of three planes is consistent if it has one or more solutions. A system of three planes is inconsistent if it has no solution.

3 scenarios of consistent solutions:

Intersect at 1 Point	Intersect in a Line	Infinite Solutions
		
Normals are not parallel and not coplanar.	Normals are not parallel but they are coplanar.	Normals are parallel and equations are scalar multiples of eachother.

Normals are parallel if each one is a scalar multiple of the others.

Normals are coplanar if $\vec{n}_1 \cdot \vec{n}_2 \times \vec{n}_3 = 0$

Example 2: For each set of planes, describe the number of solutions and how the planes intersect.

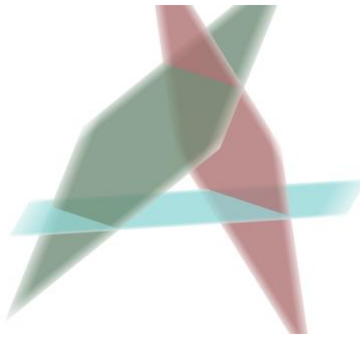
a) π_1: $2x + y + 6z - 7 = 0$
π_2: $3x + 4y + 3z + 8 = 0$
π_3: $x - 2y - 4z - 9 = 0$

https://www.geogebra.org/3d/mudhuneu

b) π_4: $x - 5y + 2z - 10 = 0$
π_5: $x + 7y - 2z + 6 = 0$
π_6: $8x + 5y + z - 20 = 0$

https://www.geogebra.org/3d/rvmqunqn

3 scenarios of inconsistent solutions:

2 planes are parallel and the third intersects both of the parallel planes	The planes intersect in pairs	The planes are parallel and at least two are distinct
		
Two normals are parallel but the third is not	Normals are not parallel but they are coplanar.	Normals are parallel but the equations are not scalar multiples of each other

a) π_1: $3x + y - 2z = 12$
π_2: $3x - 5y + z = 8$
π_3: $12x + 4y - 8z = -4$

https://www.geogebra.org/3d/kd3vjrbb

b) π_4: $x + 3y - z = -10$
π_5: $2x + y + z = 8$
π_6: $x - 2y + 2z = -4$

https://www.geogebra.org/3d/rnycrtv3

c) π_7: $4x - 2y + 6z = 35$
π_8: $-10x + 5y - 15z = 20$
π_9: $6x - 3y + 9z = -50$

https://www.geogebra.org/3d/r3ysp3gm