a) Complete Pascal's Triangle
b) What patterns do you notice in Pascal's Triangle?

**Main Pattern:**

*Each term in Pascal's Triangle is the sum of the two terms directly above it. The first and last terms in each row are 1 since the only term immediately above them is always a 1.*

Other Patterns:

- sum of each row is a power of 2 (sum of nth row is $2^n$, begin count at 0)
- symmetrical down the middle

c) Expand each of the following binomials

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a + 1b$$

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

$$(a+b)^3 = (a+b)(a+b)(a+b) = (a+b)(a^2 + 2ab + b^2) = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^3 + 4ab^2 + b^4$$
Blaise Pascal (French Mathematician) discovered a pattern in the expansion of \((a+b)^n\).... which patterns do you notice?

The coefficients in the expansion of \((a + b)^n\) can be found in row \(n\) of Pascal's triangle.

In each expansion, the exponents of \(a\) start at \(n\) and decrease by 1 down to zero, while the exponents of \(b\) start at zero and increase by 1 up to \(n\).

In each term, the sum of the exponents of \(a\) and \(b\) is always \(n\).
Example 1

Expand using the Binomial Theorem:

a) \((a + b)^6\)

\[= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6\]

b) \((2x - 3)^5\)

\[= 1(2x)^5 + 5(2x)^4(-3)^1 + 10(2x)^3(-3)^2 + 10(2x)^2(-3)^3 + 5(2x)(-3)^4 + 1(-3)^5\]

\[= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243\]
c) \((2x + 3y^2)^5\)

\[
= 1(3x)^5(3y^2)^0 + 5(3x)^4(3y^2)^1 + 10(3x)^3(3y^2)^2 \\
+ 10(3x)^2(3y^2)^3 + 5(3x)^1(3y^2)^4 + 1(3x)^0(3y^2)^5 \\
= 1(3x)(x^5)(1) + 5(16)(x^4)(3)(y^2) + 10(8)(x^3)(9)(y^4) \\
+ 10(4)(x^2)(27)(y^6) + 5(3)(x)(81)(9)(y^8) + 1(1)(243)(y^{10}) \\
= 32x^5 + 240x^4y^2 + 720x^3y^4 + 1080x^2y^6 + 810xy^8 + 243y^{10}
\]

d) \(\left(\frac{y}{2} - y^2\right)^4\)

\[
= 1\left(\frac{y}{2}\right)^4(-y)^0 + 4\left(\frac{y}{2}\right)^3(-y)^1 + 6\left(\frac{y}{2}\right)^2(-y)^2 \\
+ 4\left(\frac{y}{2}\right)^1(-y)^3 + 1\left(\frac{y}{2}\right)^0(-y)^4 \\
= \frac{y^4}{16} + 4\left(\frac{y^3}{8}\right)(-1)(y^2) + 6\left(\frac{y^2}{4}\right)(1)(y^3) \\
+ 4\left(\frac{y}{2}\right)(-1)(y^4) + 1(1)(1)(y^5) \\
= \frac{y^4}{16} - \frac{y^5}{2} + \frac{3y^6}{8} - 2y^7 + y^8
\]
Example 2

How many terms will there be if you expand \((x + 2y)^{20}\)?

\[
20 + 1 = 21
\]

Example 3

a) What is the 2nd term in the expansion of \((x+6)^7\)

\[
= 7(x)^6(6)^1
= 42x^6
\]

b) What is the 5th term in the expansion of \((3y - 4)^8\)

\[
= 70(3y)^4(-4)^4
= -70(81)(y^4)(256)
= -1451520y^4
\]
Example 4

a) What is the coefficient of $x^3$ in the expansion of $(x + 6)^6$

\[ = 20 \cdot (x)^3 \cdot (6)^3 \]
\[ = 20 \cdot x^3 \cdot 216 \]
\[ = 4320x^3 \]

b) What is the coefficient of $y^4x^2$ in the expansion of $(y + 3x)^6$

\[ = 15 \cdot (y)^4 \cdot (3x)^2 \]
\[ = 15 \cdot y^4 \cdot (9x^2) \]
\[ = 135y^4x^2 \]

Homework: Worksheet Questions