4.6 Trigonometric Identities
part 2

DO IT NOW!

Prove: $\sin\theta\sec\theta\cot\theta = 1$

LS	RS
$= \sin\theta(\sec\theta)(\cot\theta)$	$= 1$
$\downarrow$ RI	
$= \sin\theta\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\tan\theta}\right)$	
$= \frac{\sin\theta}{\cos\theta}\left(\frac{1}{\tan\theta}\right)$ $\downarrow$ QI	
$= \tan\theta\left(\frac{1}{\tan\theta}\right)$	

$= 1$ LS=RS

The Fundamental Trig Identitites

Fundamental Trigonometric Identities		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$	$\frac{\sin\theta}{\cos\theta} = \tan\theta$	$\sin^2\theta + \cos^2\theta = 1$

Tips for Proving Complex Identities

Tips and Tricks		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
Square both sides $\csc^2\theta = \frac{1}{\sin^2\theta}$ $\sec^2\theta = \frac{1}{\cos^2\theta}$ $\cot^2\theta = \frac{1}{\tan^2\theta}$	Square both sides $\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$	Rearrange the identity $\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$

General tips for proving identities:

- **i)** Try to change everything to $\sin\theta$ or $\cos\theta$
- **ii)** If you have to fractions being added or subtracted, find a common denominator and combine the fractions
- **iii)** Use difference of squares → $1 - \sin^2\theta = (1 - \sin\theta)(1 + \sin\theta)$
- **iv)** Use the power rule → $\sin^6\theta = (\sin^2\theta)^3$

Example 1:

Prove that $\cos x = \dfrac{1}{\cos x} - \sin x \tan x$

L.S.

$= \cos x$

R.S.

$= \dfrac{1}{\cos x} - \sin x\left(\dfrac{\sin x}{\cos x}\right)$

$= \dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{1-\sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x}{\cos x}$

$= \cos x$

$LS = RS$

Example 2:

Prove that $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = \dfrac{2\tan x}{\cos x}$

L.S.

$= \dfrac{1}{1-\sin x}\dfrac{(1+\sin x)}{(1+\sin x)} - \dfrac{1}{1+\sin x}\dfrac{(1-\sin x)}{(1-\sin x)}$

$= \dfrac{(1+\sin x)-(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$= \dfrac{1+\sin x - 1 + \sin x}{1-\sin^2 x}$

$= \dfrac{2\sin x}{\cos^2 x}$

R.S.

$= \dfrac{2\tan x}{\cos x}$

$= \dfrac{2\left(\frac{\sin x}{\cos x}\right)}{\frac{\cos x}{1}}$

$= \dfrac{2\sin x}{\cos x} \cdot \dfrac{1}{\cos x}$

$= \dfrac{2\sin x}{\cos^2 x}$

$LS = RS$

Example 3:

Prove that $(sin\theta + cos\theta)^2 + (sin\theta - cos\theta)^2 = 2$

Let $x = \sin\theta$

$y = \cos\theta$

L.S.	R.S.
$= (x+y)^2 + (x-y)^2$	$= 2$
$= x^2 + 2xy + y^2 + x^2 - 2xy + y^2$	
$= 2x^2 + 2y^2$	
$= 2(x^2+y^2)$	
$= 2(\sin^2\theta + \cos^2\theta)$	
$= 2(1)$	
$= 2$	

LS = RS

Example 4:

Prove that $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

L.S.	R.S.
$= \dfrac{\sin x}{\cos x}\dfrac{(1+\sin x)}{(1+\sin x)} + \dfrac{\cos x}{1+\sin x}\dfrac{(\cos x)}{(\cos x)}$	$= \sec x$
$= \dfrac{\sin x(1+\sin x) + \cos x(\cos x)}{\cos x(1+\sin x)}$	$= \dfrac{1}{\cos x}$
$= \dfrac{\sin x + (\sin^2 x + \cos^2 x)}{\cos x(1+\sin x)}$	
$= \dfrac{\sin x + 1}{\cos x(1+\sin x)}$	
$= \dfrac{1}{\cos x}$	