4.6 Trigonometric Identities
part 1

Identity: *an equation that is always true, regardless of the value of the variable.*

The Fundamental Trig Identitites

Fundamental Trigonometric Identities		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
$csc\,\theta = \frac{1}{sin\,\theta}$ $sec\,\theta = \frac{1}{cos\,\theta}$ $cot\,\theta = \frac{1}{tan\,\theta}$	$\frac{sin\,\theta}{cos\,\theta} = tan\,\theta$	$sin^2\theta + cos^2\theta = 1$

Tips for Proving Complex Identities

Tips and Tricks		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
Square both sides $csc^2\,\theta = \frac{1}{sin^2\,\theta}$ $sec^2\,\theta = \frac{1}{cos^2\,\theta}$ $cot^2\,\theta = \frac{1}{tan^2\,\theta}$	Square both sides $\frac{sin^2\,\theta}{cos^2\,\theta} = tan^2\,\theta$	Rearrange the identity $sin^2\theta = 1 - cos^2\theta$ $cos^2\theta = 1 - sin^2\theta$
General tips for proving identities: **i)** Try to change everything to $sin\,\theta$ or $cos\,\theta$ **ii)** If you have to fractions being added or subtracted, find a common denominator and combine the fractions **iii)** Use difference of squares → $1 - sin^2\,\theta = (1 - sin\,\theta)(1 + sin\,\theta)$ **iv)** Use the power rule → $sin^6\,\theta = (sin^2\,\theta)^3$		

Example 1:

Prove that $\frac{cos\theta\,tan\theta}{sin\theta} = 1$

L.S.	R.S.
$= \frac{\cos\theta \tan\theta}{\sin\theta}$	$= 1$
$= \frac{\cancel{\cos\theta}\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)}{\sin\theta}$	
$= \frac{\sin\theta}{\sin\theta}$	
$= 1$	

Q.I.

LS = RS

Example 2: Prove that $tan^2\theta + 1 = sec^2\theta$

L.S.	R.S.
$= \tan^2\theta + 1$ ↓ Q.I.	$= \sec^2\theta$ ↓ R.I.
$= \frac{\sin^2\theta}{\cos^2\theta} + 1$	
$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}$ get a common denom	$= \frac{1}{\cos^2\theta}$
$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$ ↓ P.I.	
$= \frac{1}{\cos^2\theta}$	
LS = RS	

Example 3: Prove that $\cos^2 x = (1-\sin x)(1+\sin x)$

L.S.	R.S.
$= \cos^2 x$ ↓ P.I.	$= (1-\sin x)(1+\sin x)$ ↓ DOS
$= 1-\sin^2 x$	$= 1-\sin^2 x$
LS = RS	

Example 4:

Prove that $1 - cos^2\theta = sin\theta\ cos\theta\ tan\theta$

L.S.	R.S.
$= 1 - \cos^2\theta$ ↓ P.I.	$= \sin\theta(\cos\theta)(\tan\theta)$ ↓ QI
$= \sin^2\theta$	$= \sin\theta(\cos\theta)\left(\frac{\sin\theta}{\cos\theta}\right)$
	$= \sin\theta(\sin\theta)$
	$= \sin^2\theta$

LS = RS

Example 5:

Prove that $sin^6\theta = (1 - cos^2\theta)^3$

L.S.	R.S.
$= \sin^6\theta$	$= (1 - \cos^2\theta)^3$ ↓ P.I.
	$= (\sin^2\theta)^3$
	$= \sin^6\theta$

LS = RS

Example 6:

Prove that $\dfrac{\sin^2 x}{1-\cos x} = 1+\cos x$

L.S.

$= \dfrac{\sin^2 x}{1-\cos x}$

↓ P.I.

$= \dfrac{1-\cos^2 x}{1-\cos x}$

↓ DOS

$= \dfrac{(1-\cos x)(1+\cos x)}{1-\cos x}$

$= 1+\cos x$

R.S.

$= 1+\cos x$

LS = RS