Chapter 1 PRE-TEST REVIEW – Polynomial Functions
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SOLUTIONS

Section 1: 1.1 Power Functions

1) State the degree and the leading coefficient of each polynomial

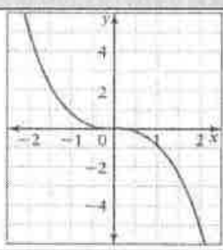
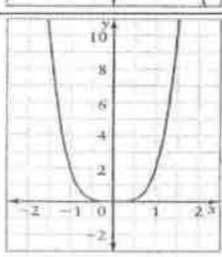
Polynomial	Degree	Leading Coefficient
$y = 2x^3 + 3x - 1$	3	2
$y = 5x - 6$	1	5
$y = x^3 - 2x^2 - 5x^4 + 3$	4	-5
$y = -3x^5 + 2x^3 - x - 1$	5	-3
$y = 21 - 2x + 4x^2 - 6x^3$	3	-6

2) Match each function to its end behavior

$y = 3x^7$ $\qquad$ $y = -\frac{1}{2}x^3$ $\qquad$ $y = 2x^4$ $\qquad$ $y = -0.25x^6$

End Behaviour	Functions
Q3 to Q1	$y = 3x^7$
Q2 to Q4	$y = \frac{-1}{2}x^3$
Q2 to Q1	$y = 2x^4$
Q3 to Q4	$y = -0.25x^6$

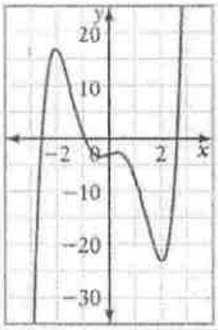
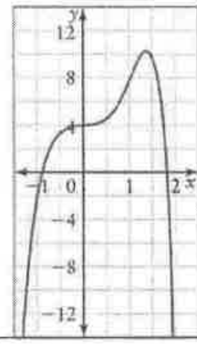
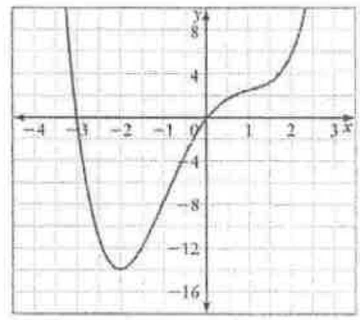
3) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	Odd	−	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$	Point about the origin	Q2 → Q4
	Even	+	D: $(-\infty, \infty)$ R: $[0, \infty)$	Line about the y-axis	Q2 → Q1

Section 2: 1.2 Characteristics of Polynomial Functions

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

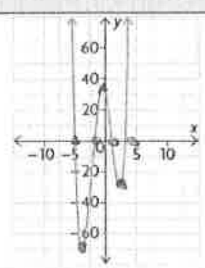
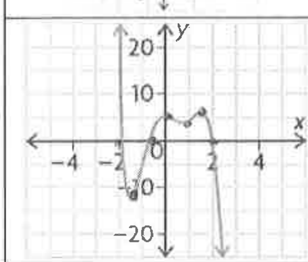
A) $g(x) = 0.5x^4 - 3x^2 + 5x$ **B)** $h(x) = x^5 - 7x^3 + 2x - 3$ **C)** $p(x) = -x^6 + 5x^3 + 4$

		
B	C	A

5) Complete the following table

Equation	Degree	Sign of Leading Coefficient	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = 6x^3 + 2x$	3	+	Q3→Q1	2, 0	3, 2, 1
$g(x) = -20x^6 - 5x^3 + x^2 - 17$	6	−	Q3→Q4	5, 3, 1	6, 5, 4, 3, 2, 1, 0
$p(x) = 22x^4 - 4x^3 + 3x^2 - 2x + 2$	4	+	Q2→Q1	3, 1	4, 3, 2, 1, 0
$h(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$	5	−	Q2→Q4	4, 2, 0	5, 4, 3, 2, 1

6) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q2→Q1	None	3	4	4
	−	Odd	Q2→Q4	None	4	3	5

7) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) fifth differences = -60 Degree 5

$-60 = a(5!)$

$-60 = 120a$

$\boxed{a = -\frac{1}{2}}$

b) third differences = 42 Degree 3

$42 = a(3!)$

$42 = 6a$

$\boxed{a = 7}$

8) For each function, find the value of the constant finite differences.

a) $g(x) = 0.5x^4 - 3x^2 + 5x$

Finite Differences $= 0.5(4!)$

$= 0.5(24)$

$\boxed{= 12}$

b) $h(x) = x^5 - 7x^3 + 2x - 3$

Finite Differences $= 1(5!)$

$\boxed{= 120}$

9) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
−3	124
−2	41
−1	8
0	1
1	−4
2	−31
3	−104
4	−247

1st: -83, -33, -7, -5, -27, -73, -143

2nd: 50, 26, 2, -22, -46, -70

3rd: -24, -24, -24, -24, -24

$\boxed{\text{Degree} = 3}$

$-24 = a(3!)$

$-24 = 6a$

$\boxed{a = -4}$

b)

x	y
−2	−229
−1	−5
0	3
1	−7
2	−53
3	−129
4	35
5	1213

1st: 224, 8, -10, -46, -76, 164, 1178

2nd: -216, -18, -36, -30, 240, 1014

3rd: 198, -18, 6, 270, 774

4th: -216, 24, 264, 504

5th: 240, 240, 240

$\boxed{\text{Degree} = 5}$

$240 = a(5!)$

$240 = 120a$

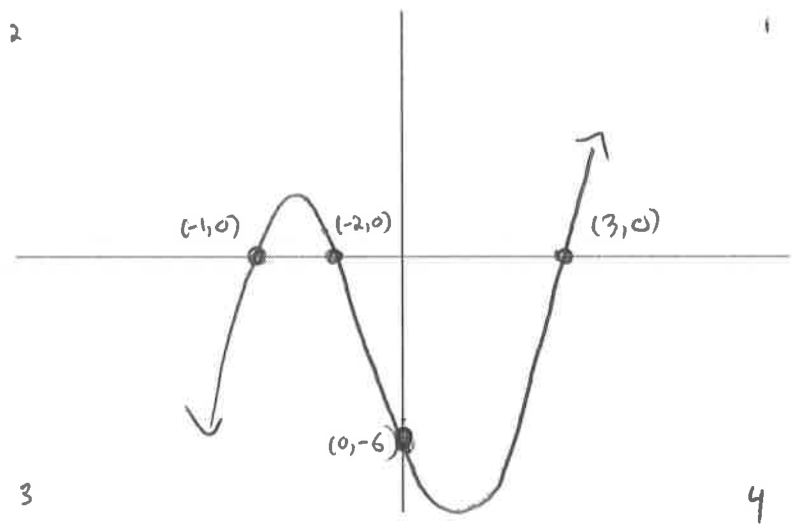
$\boxed{a = 2}$

Section 3: 1.3 Factored Form Polynomial Functions

10) For each function, complete the chart and sketch a possible graph of the function labelling key points.

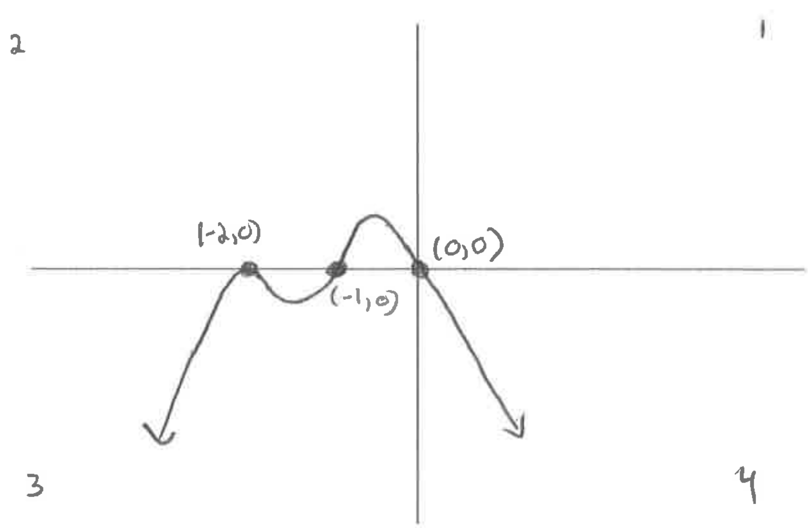
a) $f(x) = (x+1)(x-3)(x+2)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$(1)(1)(1)$ $=1$	Q3 → Q1	$(-1,0)$ $(3,0)$ $(-2,0)$	$f(0)=(0+1)(0-3)(0+2)$ $=(1)(-3)(2)$ $=-6$



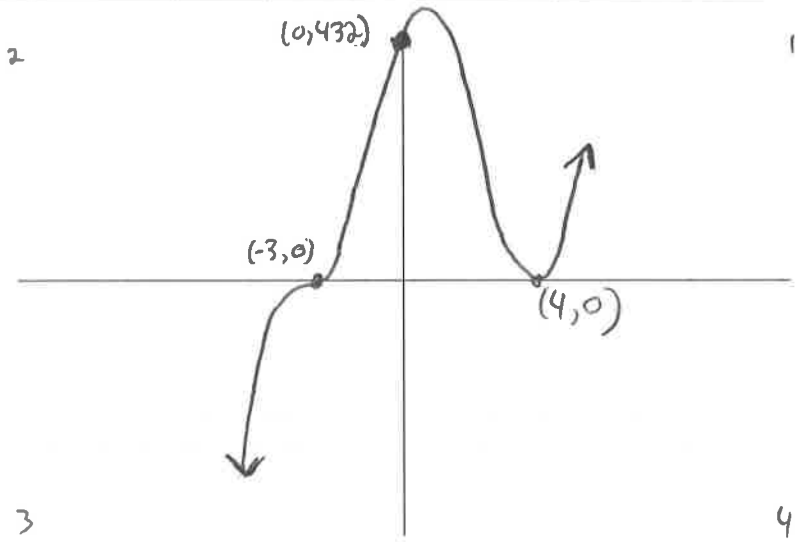
b) $g(x) = -x(x+1)(x+2)^2$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x^2)$ $= x^4$ Degree 4	$-1(1)(1)^2$ $= -1$	Q3 → Q4	$(0,0)$ $(-1,0)$ $(-2,0)$ order 2	$g(0) = -(0)(0+1)(0+2)^2$ $= -(0)(1)(4)$ $= 0$



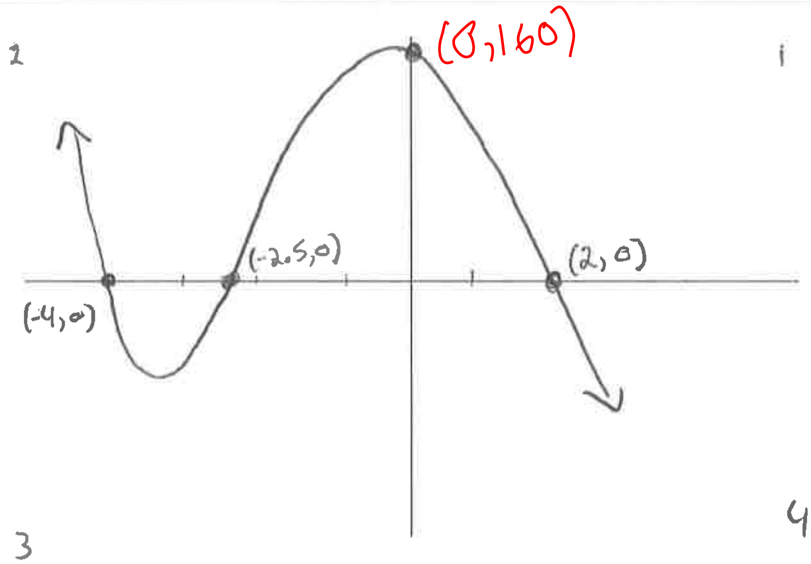
c) $h(x) = (x-4)^2(x+3)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x^2)(x^3)$ $= x^5$ Degree 5	$(1)^2(1)^3$ $= 1$	Q3 → Q1	(4,0) order 2 (-3,0) order 3	$h(0) = (0-4)^2(0+3)^3$ $= (16)(27)$ $= 432$



d) $p(x) = -4(2x+5)(x-2)(x+4)$

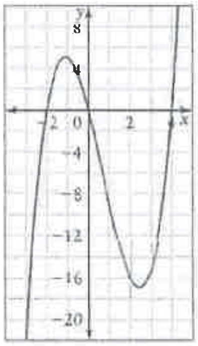
Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$-4(2)(1)(1)$ $= -8$	Q2 → Q4	$(-5/2, 0)$ (2,0) (-4,0)	$p(0) = -4[2(0)+5](0-2)(0+4)$ $= -4(5)(-2)(4)$ $= 160$



11) For each graph, state...

i) the least possible degree and the sign of the leading coefficient
ii) the x-intercepts (specify order of zero) and the factors of the function
iii) the intervals where the function is positive/negative

a)



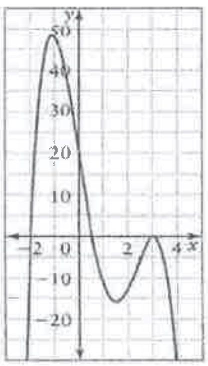
i) degree: 3
leading coefficient: POSITIVE

ii) x-intercepts: $-2, 0, 4$
factors: $(x+2), x, (x-4)$

iii)

Interval	$(-\infty,-2)$	$(-2,0)$	$(0,4)$	$(4,\infty)$
Sign	$-$	$+$	$-$	$+$

b)



i) degree: 4
leading coefficient: NEGATIVE

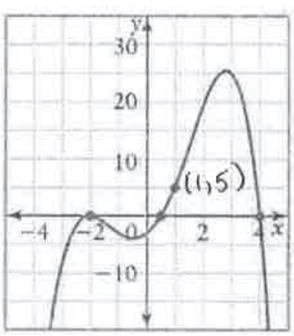
ii) x-intercepts: $-2, \frac{1}{2}, 3$ (order 2)
factors: $(x+2), (2x-1), (x-3)^2$

iii)

Interval	$(-\infty,-2)$	$(-2,0.5)$	$(0.5,3)$	$(3,\infty)$
Sign	$-$	$+$	$-$	$-$

12) Write the equation of each of the following functions:

a)



$$f(x) = K(x+2)^2(2x-1)(x-4)$$
$$5 = K(1+2)^2[2(1)-1](1-4)$$
$$5 = K(9)(1)(-3)$$
$$5 = -27K$$
$$K = -\frac{5}{27}$$

$$\boxed{f(x) = -\frac{5}{27}(x+2)^2(2x-1)(x-4)}$$

b) The quartic function has zeros at -3, -1, and 2 (order 2) and passes through the point (1, 4)

$$g(x) = K(x+3)(x+1)(x-2)^2$$
$$4 = K(1+3)(1+1)(1-2)^2$$
$$4 = K(4)(2)(1)$$
$$4 = 8K$$
$$K = \frac{1}{2}$$

$$\boxed{g(x) = \frac{1}{2}(x+3)(x+1)(x-2)^2}$$

Section 4: 1.4 Transformations of Polynomial Functions

12) Write an equation for the function that results from the given transformations.

a) The function $f(x) = x^4$ is compressed vertically by a factor of $\frac{3}{5}$, stretched horizontally by a factor of 2, reflected horizontally in the y-axis, and translated 1 unit up and 4 units to the left.

$$g(x) = \frac{3}{5}\left[-\frac{1}{2}(x+4)\right]^4 + 1$$

b) The function $f(x) = x^3$ is compressed horizontally by a factor of $\frac{1}{4}$, stretched vertically by a factor of 5, reflected vertically in the x-axis, and translated 2 units to the left and 7 units up.

$$g(x) = -5\left[4(x+2)\right]^3 + 7$$

13) Identify the a, k, d and c values and explain what transformation is occurring to the parent function for $g(x) = 2[-4(x+7)]^4 - 1$

$a = 2$; vertical stretch by a factor of 2 $(2y)$

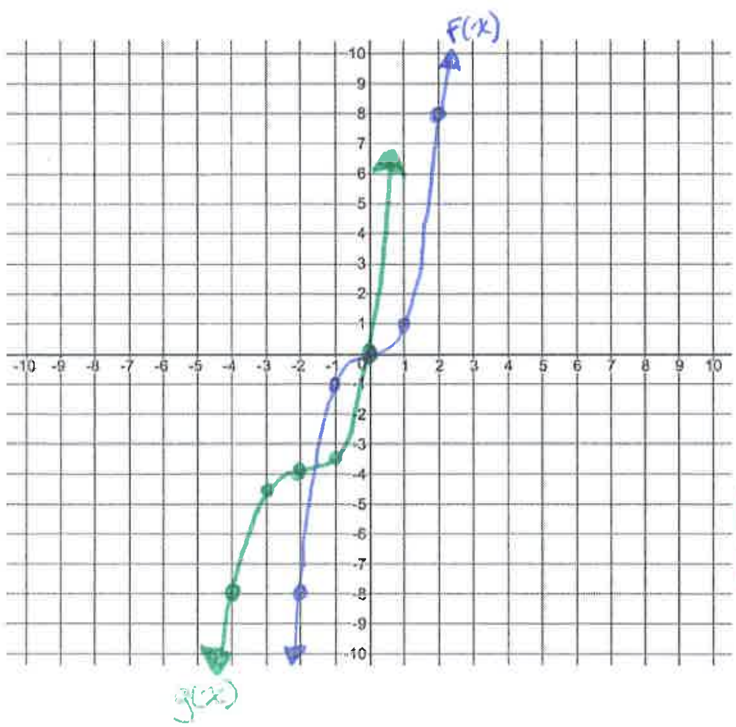
$k = -4$; horizontal reflection and horizontal compression bafo $\frac{1}{4}$ $\left(\frac{x}{-4}\right)$

$d = -7$; shift left 7 units $(x-7)$

$c = -1$; shift down 1 unit. $(y-1)$

14) For the following questions, use the key points of the parent function to perform transformations. Graph the parent and transformed function. Write the equation of the transformed function.

a) $f(x) = x^3$ $\qquad g(x) = \frac{1}{2}f(x+2) - 4$



$\boxed{f(x)}$

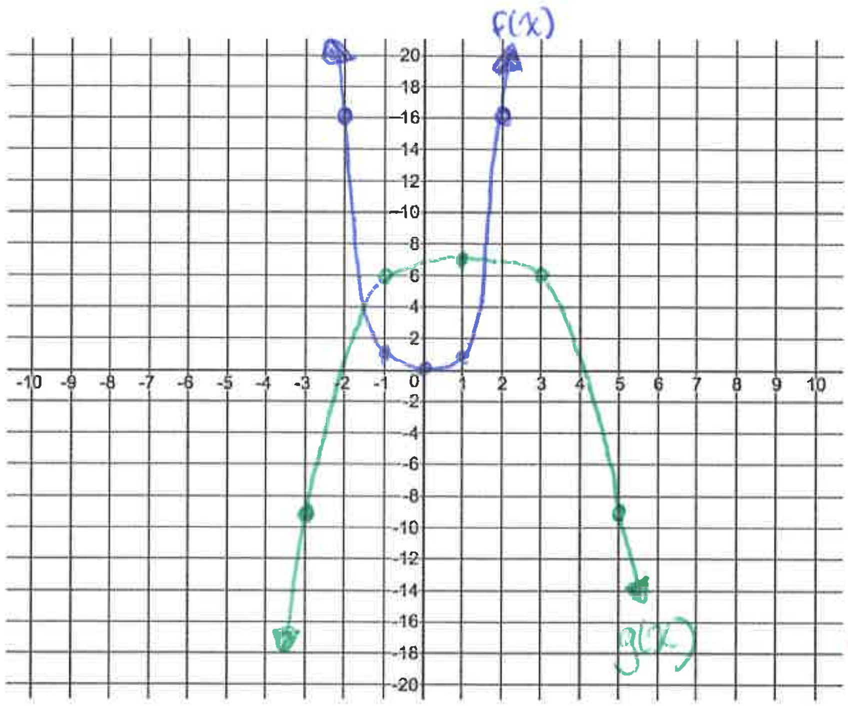
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$\boxed{g(x)}$

$x-2$	$\frac{y}{2}-4$
-4	-8
-3	-4.5
-2	-4
-1	-3.5
0	0

$\boxed{Eq^n: g(x) = \frac{1}{2}(x+2)^3 - 4}$

b) $f(x) = x^4$ $\qquad$ $g(x) = -f\left[\frac{1}{2}(x-1)\right] + 7$



f(x)

x	y
-2	16
-1	1
0	0
1	1
2	16

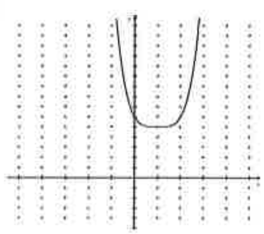
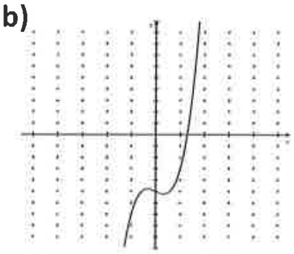
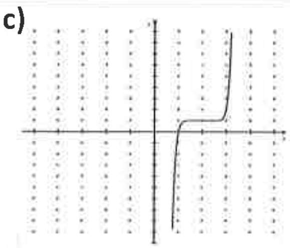
g(x)

$2x+1$	$-y+7$
-3	-9
-1	6
1	7
3	6
5	-9

$$\boxed{\text{Eq}^n: -\left[\tfrac{1}{2}(x-1)\right]^4 + 7}$$

Section 5: 1.5 Symmetry

`5) Circle all that apply for each function

a) 	No symmetry Even function Odd function Line Symmetry Point Symmetry	**d)** $f(x) = 3x^6 + 2x^2 - 5$	No symmetry Even function Odd function Line Symmetry Point Symmetry
b) 	No symmetry Even function Odd function Line Symmetry Point Symmetry	**e)** $f(x) = x^3 - 4x^2 + 1$	No symmetry Even function Odd function Line Symmetry Point Symmetry
c) 	No symmetry Even function Odd function Line Symmetry Point Symmetry	**f)** $f(x) = x^4 + 5x$	No symmetry Even function Odd function Line Symmetry Point Symmetry

All cubics have point symmetry.

16) Consider the polynomial function $f(x) = -3x^4 + 6x^2 - 10$

a) Show algebraically whether *f* is even, odd or neither.

$f(-x) = -3(-x)^4 + 6(-x)^2 - 10$

$f(-x) = -3(-1)^4(x)^4 + 6(-1)^2(x)^2 - 10$

$f(-x) = -3x^4 + 6x^2 - 10$

$\therefore f(-x) = f(x)$

Since $f(-x) = f(x)$, it is an even function.

b) For what finite difference will *f* give a constant value, and what will that constant value be?

It is degree 4, so the 4^{th} differences will be constant.

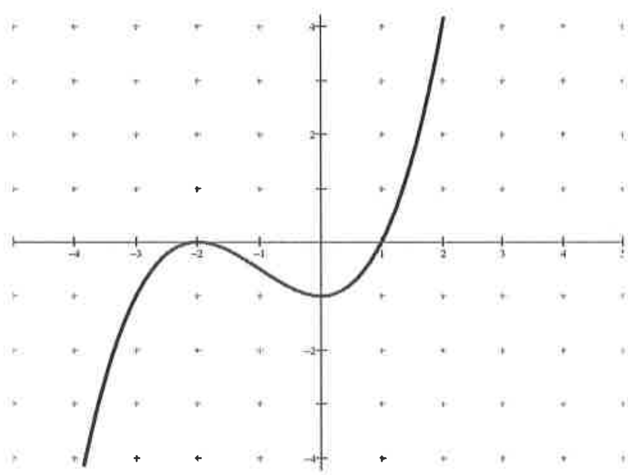
Finite differences $= -3(4!)$
$= -72$

c) What are the maximum and minimum number of zeros the above polynomial could have?

$\boxed{\text{Min zeros} = 0 \quad \text{Max zeros} = 4}$

Possible zeros for degree 4 are 4, 3, 2, 1, or 0.

17) Use the given graph to state:



a) x-intercepts -2 (order 2), and 1

b) number of turning points 2

c) least possible degree 3

d) any symmetry present; even or odd function?

Point symmetry. Not an even or odd function.

e) the intervals where $f(x) < 0$

$f(x) < 0$ when: $x < -2$ or $-2 < x < 1$

$f(x) < 0$ when $x \in (-\infty, -2) \cup (-2, 1)$