

Unit 2 Pre-Test Review – Factor Theorem and Inequalities

MHF4U

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SOLUTIONS

Section 1: 2.1 - Long and Synthetic Division / Remainder Theorem

1) What is the remainder when  $x^4 - 4x^2 - 2x + 3$  is divided by  $x + 1$ ? Do not divide. Support your answer with an explanation.

$$\begin{aligned} P(-1) &= (-1)^4 - 4(-1)^2 - 2(-1) + 3 \\ &= 1 - 4 + 2 + 3 \\ &= 2 \end{aligned}$$

Based on the remainder theorem, the remainder will be 2.

2) Is  $x - 3$  a factor of the polynomial  $3x^2 - 8x - 3$ ? Do not divide. Support your answer with an explanation.

$$\begin{aligned} P(3) &= 3(3)^2 - 8(3) - 3 \\ &= 27 - 24 - 3 \\ &= 0 \end{aligned}$$

Yes because the remainder is 0.

3) Divide  $\frac{f(x)}{g(x)}$  and state the answer in quotient form. Use synthetic division where possible.

a)  $f(x) = x^4 - 4x^2 - 2x + 3, g(x) = x - 2$       b)  $f(x) = x^5 - x^4 + 2x^3 + 3x - 2, g(x) = x^2 + 2$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -4 & -2 & 3 \\ & \downarrow & 2 & 4 & 0 & -4 & + \\ \hline x & 1 & 2 & 0 & -2 & -1 \\ & x^3 & x^2 & x & R & \end{array}$$

$$\frac{x^4 - 4x^2 - 2x + 3}{x - 2} = x^3 + 2x^2 - 2 - \frac{1}{x - 2}$$

$$\begin{array}{r} x^3 - 1x^2 + 0x + 2 \\ x^2 + 0x + 2 \overline{) x^5 - x^4 + 2x^3 + 0x^2 + 3x - 2} \\ \underline{x^5 + 0x^4 + 2x^3} \phantom{+ 0x^2 + 3x - 2} \\ -1x^4 + 0x^3 + 0x^2 \phantom{+ 3x - 2} \\ \underline{-1x^4 + 0x^3 - 2x^2} \phantom{+ 3x - 2} \\ 0x^3 + 2x^2 + 3x \phantom{- 2} \\ \underline{0x^3 + 0x^2 + 0x} \phantom{- 2} \\ 2x^2 + 3x - 2 \\ \underline{2x^2 + 0x + 4} \\ R = 3x - 6 \end{array}$$

$$\frac{x^5 - x^4 + 2x^3 + 3x - 2}{x^2 + 2} = x^3 - x^2 + 2 + \frac{3x - 6}{x^2 + 2}$$

4) Perform each division. Express the answer in quotient form and write the statement that could be used to check the division.

a)  $x^3 + 9x^2 - 5x + 3$  divided by  $x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 9 & -5 & 3 \\ & \downarrow & 2 & 22 & 34 & + \\ \times & 1 & 11 & 17 & 37 \\ & x^2 & x & \# & R \end{array}$$

$$\text{Q.F.: } \frac{x^3 + 9x^2 - 5x + 3}{x - 2} = x^2 + 11x + 17 + \frac{37}{x - 2}$$

As product:

$$x^3 + 9x^2 - 5x + 3 = (x - 2)(x^2 + 11x + 17) + 37$$

b)  $12x^3 - 2x^2 + x - 11$  divided by  $3x + 1$

$$\begin{array}{r} 4x^2 - 2x + 1 \\ 3x + 1 \overline{) 12x^3 - 2x^2 + x - 11} \\ \underline{12x^3 + 4x^2} \phantom{- 11} \downarrow \\ -6x^2 + x \phantom{- 11} \downarrow \\ \underline{-6x^2 - 2x} \phantom{- 11} \\ 3x - 11 \\ \underline{3x + 1} \\ R = -12 \end{array}$$

$$\text{Q.F.: } \frac{12x^3 - 2x^2 + x - 11}{3x + 1} = 4x^2 - 2x + 1 - \frac{12}{3x + 1}$$

Product:

$$12x^3 - 2x^2 + x - 11 = (3x + 1)(4x^2 - 2x + 1) - 12$$

c)  $-8x^4 - 4x + 10x^3 - x^2 + 15$  divided by  $2x - 1$

$$\begin{array}{r} -4x^3 + 3x^2 + x - \frac{3}{2} \\ 2x - 1 \overline{) -8x^4 + 10x^3 - x^2 - 4x + 15} \\ \underline{-8x^4 + 4x^3} \phantom{- 15} \downarrow \\ 6x^3 - x^2 \phantom{- 4x + 15} \downarrow \\ \underline{6x^3 - 3x^2} \phantom{- 4x + 15} \\ 2x^2 - 4x \phantom{+ 15} \downarrow \\ \underline{2x^2 - x} \phantom{+ 15} \\ -3x + 15 \phantom{+ 15} \downarrow \\ \underline{-3x + \frac{3}{2}} \\ R = \frac{27}{2} \end{array}$$

$$\text{Q.F.: } \frac{-8x^4 + 10x^3 - x^2 - 4x + 15}{2x - 1} = -4x^3 + 3x^2 + x - \frac{3}{2} + \frac{27}{2(2x - 1)}$$

Product:

$$-8x^4 + 10x^3 - x^2 - 4x + 15 = (2x - 1)(-4x^3 + 3x^2 + x - \frac{3}{2}) + \frac{27}{2}$$

d)  $x^3 + 4x^2 - 3$  divided by  $x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & 0 & -3 \\ & \downarrow & 2 & 12 & 24 & + \\ \times & 1 & 6 & 12 & 21 \\ & x^2 & x & \# & R \end{array}$$

$$\text{Q.F.: } \frac{x^3 + 4x^2 - 3}{x - 2} = x^2 + 6x + 12 + \frac{21}{x - 2}$$

Product:

$$x^3 + 4x^2 - 3 = (x - 2)(x^2 + 6x + 12) + 21$$

5) Determine the value of  $k$  such that when  $f(x) = x^4 + kx^3 - 3x - 5$  is divided by  $x - 3$ , the remainder is  $-10$ .

$$f(3) = (3)^4 + k(3)^3 - 3(3) - 5$$

$$-10 = 81 + 27k - 9 - 5$$

$$-10 = 27k + 67$$

$$-77 = 27k$$

$$k = \frac{-77}{27}$$

### Section 2: 2.2 - Factor Theorem

6) Suppose the cubic polynomial  $8x^3 + mx^2 + nx - 6$  has both  $2x + 3$  and  $x - 1$  as factors. Find  $m$  and  $n$ . Do not divide.

$$0 = 8\left(-\frac{3}{2}\right)^3 + m\left(-\frac{3}{2}\right)^2 + n\left(-\frac{3}{2}\right) - 6$$

$$0 = -27 + \frac{9}{4}m - \frac{3}{2}n - 6$$

$$33 = \frac{9}{4}m - \frac{3}{2}n$$

$$\textcircled{1} 132 = 9m - 6n$$

$$0 = 8(1)^3 + m(1)^2 + n(1) - 6$$

$$0 = 8 + m + n - 6$$

$$\textcircled{2} -2 = m + n$$

$$\textcircled{1} 132 = 9m - 6n$$

$$6 \times \textcircled{2} -12 = 6m + 6n +$$

$$120 = 15m$$

$$m = 8$$

sub  $m=8$  into  $\textcircled{2}$

$$-2 = 8 + n$$

$$-10 = n$$

7) Factor each of the following

a)  $x^3 - 4x^2 + x + 6$

Possible factors:  $\pm 1, \pm 2, \pm 3, \pm 6$

$f(2) = 0 \Rightarrow x-2$  is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 1 & 6 \\ & \downarrow & & & \\ x & 1 & -2 & -3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$x^3 - 4x^2 + x + 6 = (x-2)(x^2 - 2x - 3)$$

$$= (x-2)(x-3)(x+1)$$

b)  $3x^3 - 5x^2 - 26x - 8$

Possible factors:  $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm 8$

$f(-2) = 0 \Rightarrow x+2$  is a factor

$$\begin{array}{r|rrrr} -2 & 3 & -5 & -26 & -8 \\ & \downarrow & & & \\ x & 3 & -11 & -4 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$\begin{array}{r} P \\ \frac{4}{1} \\ \frac{-12}{3} \\ \frac{-12}{3} \\ \frac{1}{3} \\ -11 \\ 5 \end{array}$$

$$3x^3 - 5x^2 - 26x - 8 = (x+2)(3x^2 - 11x - 4)$$

$$= (x+2)(x-4)(3x+1)$$

$$\text{DOC: } a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\text{d) } x^3 - 64 = x^3 - 4^3$$

$$= (x-4)(x^2 + 4x + 4^2)$$

$$= (x-4)(x^2 + 4x + 16)$$

$$\text{c) } -4x^3 - 4x^2 + 16x + 16$$

$$= -4(x^3 + x^2 - 4x - 4)$$

$$= -4[x^2(x+1) - 4(x+1)]$$

$$= -4(x+1)(x^2 - 4)$$

$$= -4(x+1)(x-2)(x+2)$$

### Section 3: 2.3&2.6 – Factoring to Solve Equations and Inequalities

8) Determine the real roots of each equation.

$$\text{a) } (5x^2 + 20)(3x^2 - 48) = 0$$

$$\checkmark$$
$$5x^2 + 20 = 0$$

$$5x^2 = -20$$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

∴ No solutions

$$\checkmark$$
$$3x^2 - 48 = 0$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$x = \pm\sqrt{16}$$

$$x_1 = 4 \quad x_2 = -4$$

$$\text{b) } (2x^2 - x - 13)(x^2 + 1) = 0$$

NOT Factorable  
So use QF

$$\checkmark$$
$$x^2 + 1 = 0$$

$$x = \pm\sqrt{-1}$$

∴ No solutions

$$x = \frac{1 \pm \sqrt{1 - 4(2)(-13)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{105}}{4}$$

$$x_1 = \frac{1 + \sqrt{105}}{4}$$

$$x_2 = \frac{1 - \sqrt{105}}{4}$$

9) Solve the following polynomial equations.

a)  $2x^3 + 1 = x^2 + 2x$

$$2x^3 - x^2 - 2x + 1 = 0$$

$$x^2(2x-1) - 1(2x-1) = 0$$

$$(2x-1)(x^2-1) = 0$$

$$(2x-1)(x-1)(x+1) = 0$$

$$\boxed{x_1 = \frac{1}{2}} \quad \boxed{x_2 = 1} \quad \boxed{x_3 = -1}$$

b)  $x^3 + 6x^2 + 11x + 6 = 0$

Possible factors:  $\pm 1, \pm 2, \pm 3, \pm 6$

$f(-1) = 0$ ,  $\therefore x+1$  is a factor

$$\begin{array}{r|rrrrr} -1 & 1 & 6 & 11 & 6 & \\ & & -1 & -5 & -6 & + \\ \hline x & 1 & 5 & 6 & 0 & \\ & & x^2 & x & \# & R \end{array}$$

$$(x+1)(x^2+5x+6) = 0$$

$$(x+1)(x+2)(x+3) = 0$$

$$\boxed{x_1 = -1} \quad \boxed{x_2 = -2} \quad \boxed{x_3 = -3}$$

c)  $x^5 - 4x^3 - x^2 + 4 = 0$

$$x^3(x^2-4) - 1(x^2-4) = 0$$

$$(x^2-4)(x^3-1) = 0$$

$$(x-2)(x+2)(x-1)(x^2+1x+1) = 0$$

$$\boxed{x_1 = 2} \quad \boxed{x_2 = -2} \quad \boxed{x_3 = 1}$$

No solutions  
 $b^2 - 4ac = -3$

d)  $3x^3 + 2x^2 - 11x - 10 = 0$

Possible factors:  $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{3}, \pm 10, \pm \frac{10}{3}$

$f(-1) = 0$ ,  $\therefore x+1$  is a factor

$$\begin{array}{r|rrrrr} -1 & 3 & 2 & -11 & -10 & \\ & & -3 & 1 & 10 & + \\ \hline x & 3 & -1 & -10 & 0 & \\ & & x^2 & x & \# & R \end{array}$$

$$(x+1)(3x^2-x-10) = 0$$

$$(x+1)(x-2)(3x+5) = 0$$

$$\boxed{x_1 = -1} \quad \boxed{x_2 = 2} \quad \boxed{x_3 = -\frac{5}{3}}$$

P

$$\begin{array}{c} -30 \\ \hline \begin{array}{l} \frac{-2}{1} \\ \frac{-6}{3} \\ \frac{5}{3} \\ -1 \\ 5 \end{array} \end{array}$$

10) Solve the following polynomial inequalities. (Refer to #9 where you factored the polynomials)

a)  $2x^3 + 1 < x^2 + 2x$

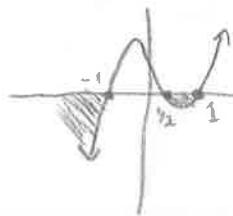
$2x^3 - x^2 - 2x + 1 < 0$

$(2x-1)(x-1)(x+1) < 0$

x-int at  $x = -1, 1/2, 1$

+ L.C., odd degree

Q3 → Q1



**Solution:**  
 when  $x < -1$  or  $0.5 < x < 1$   
 OR  
 when  $x \in (-\infty, -1) \cup (0.5, 1)$

	$-\infty$	$-1$	$0.5$	$1$	$\infty$
$2x-1$		-	-	+	+
$x-1$		-	-	-	+
$x+1$		-	+	+	+
overall		⊖	+	⊖	+

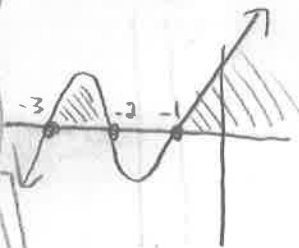
b)  $x^3 + 6x^2 + 11x + 6 > 0$

$(x+1)(x+2)(x+3) > 0$

x-int at  $x = -3, -2, -1$

+ L.C., odd degree

Q3 → Q1



**Solution:**

when  $-3 < x < -2$  OR  $x > -1$

when  $x \in (-3, -2) \cup (-1, \infty)$

	$-\infty$	$-3$	$-2$	$-1$	$\infty$
$x+1$		-	-	-	+
$x+2$		-	-	+	+
$x+3$		-	+	+	+
overall		-	⊕	-	⊕

11) Where is the polynomial  $y = 8x^3 + 1$  positive? Justify your solution.

SOC

$8x^3 + 1 > 0$

$(2x+1)[(2x)^2 - (2x)(1) + (1)^2] > 0$

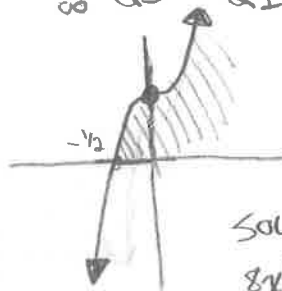
$(2x+1)(4x^2 - 2x + 1) > 0$

$x = -\frac{1}{2}$

check  $b^2 - 4ac = (-2)^2 - 4(4)(1) = -12$   
 ∴ NO ROOTS

Positive L.C. and odd degree.

∴ Q3 → Q1.



**Solution:**

$8x^3 + 1 > 0$  when  $x > -\frac{1}{2}$

when  $x \in (-\frac{1}{2}, \infty)$

12) Solve  $6x^3 + 13x^2 - 41x + 12 \leq 0$  using a sign chart.

Possible zeros:  $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm \frac{3}{2}, \pm 4, \pm \frac{4}{3}, \pm 6, \pm 12$

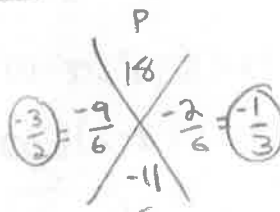
$f(-4) = 0$ ; ∴  $x+4$  is a factor

	$-\infty$	$-4$	$\frac{1}{3}$	$\frac{3}{2}$	$\infty$
$x+4$		-	+	+	+
$2x-3$		-	-	-	+
$3x-1$		-	-	+	+
overall		⊖	+	⊖	+

$(x+4)(6x^2 - 11x + 3) \leq 0$

$(x+4)(2x-3)(3x-1) \leq 0$

x-int at  $x = -4, \frac{1}{3}, \frac{3}{2}$



	$-\infty$	$-4$	$\frac{1}{3}$	$\frac{3}{2}$	$\infty$
$x+4$		-	+	+	+
$2x-3$		-	-	-	+
$3x-1$		-	-	+	+
overall		⊖	+	⊖	+

**Solution:**

when  $x \leq -4$  or  $\frac{1}{3} \leq x \leq \frac{3}{2}$

when  $x \in (-\infty, -4] \cup [\frac{1}{3}, \frac{3}{2}]$

**Section 4: 2.4 – Families of Polynomials**

13) Find the equation for the family of quartic polynomials that have real roots of 3 (order 2) and  $2 \pm \sqrt{2}$ .

Factors:  
 $x = 3$   
 $x - 3 = 0$   
 $(x - 3)$  order 2

$$x = 2 \pm \sqrt{2}$$

$$x - 2 = \pm \sqrt{2}$$

$$(x - 2)^2 = 2$$

$$x^2 - 4x + 4 = 2$$

$$x^2 - 4x + 2 = 0$$

$$(x^2 - 4x + 2)$$

Equation for family:

$$y = k(x - 3)^2(x^2 - 4x + 2)$$

14) A family of cubic polynomials has roots of -2, -3 and -5. Find the member of this family that passes through the point (2, -35). What is this polynomials y-intercept?

$$f(x) = k(x + 2)(x + 3)(x + 5)$$

$$-35 = k(2 + 2)(2 + 3)(2 + 5)$$

$$-35 = k(4)(5)(7)$$

$$-35 = 140k$$

$$k = -\frac{1}{4}$$

$$f(0) = -\frac{1}{4}(0 + 2)(0 + 3)(0 + 5)$$

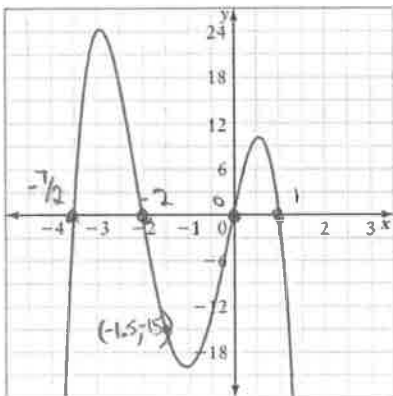
$$f(0) = -\frac{1}{4}(30)$$

$$f(0) = -\frac{15}{2}$$

The equation is  $f(x) = -\frac{1}{4}(x + 2)(x + 3)(x + 5)$ .  
 It has a y-intercept of  $(0, -\frac{15}{2})$

15) Find an equation for each of the following functions

a)



$$f(x) = k(2x + 7)(x + 2)(x)(x - 1)$$

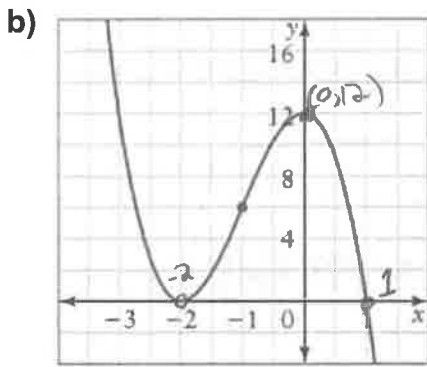
$$-15 = k[2(-1.5) + 7](-1.5 + 2)(-1.5)(-1.5 - 1)$$

$$-15 = k(4)(0.5)(-1.5)(-2.5)$$

$$-15 = 7.5k$$

$$k = -2$$

$$f(x) = -2(x)(2x + 7)(x + 2)(x - 1)$$



$$f(x) = k(x+2)^2(x-1)$$

$$12 = k(0+2)^2(0-1)$$

$$12 = k(4)(-1)$$

$$12 = -4k$$

$$k = -3$$

$$f(x) = -3(x+2)^2(x-1)$$

### ANSWER KEY

1)  $P(-1) = 2$  = remainder . This was found using remainder theorem.

2)  $P(3) = 0$ , so  $x - 3$  is a factor because remainder is 0 (Factor Theorem)

3) a)  $\frac{x^4 - 4x^2 - 2x + 3}{x-2} = x^3 + 2x^2 - 2 - \frac{1}{x-2}$     b)  $\frac{x^5 - x^4 + 2x^3 + 3x - 2}{x^2 + 2} = x^3 - x^2 + 2 + \frac{3x-6}{x^2+2}$

4) a)  $\frac{x^3 + 9x^2 - 5x + 3}{x-2} = x^2 + 11x + 17 + \frac{37}{x-2}$ ;  $x^3 + 9x^2 - 5x + 3 = (x-2)(x^2 + 11x + 17) + 37$

b)  $\frac{12x^3 - 2x^2 + x - 11}{3x+1} = 4x^2 - 2x + 1 - \frac{12}{3x+1}$ ;  $12x^3 - 2x^2 + x - 11 = (3x+1)(4x^2 - 2x + 1) - 12$

c)  $\frac{-8x^4 - 4x + 10x^3 - x^2 + 15}{2x-1} = -4x^3 + 3x^2 + x - \frac{3}{2} + \frac{27}{2(2x-1)}$ ;  $-8x^4 - 4x + 10x^3 - x^2 + 15 = (2x-1)\left(-4x^3 + 3x^2 + x - \frac{3}{2}\right) + \frac{27}{2}$

d)  $\frac{x^3 + 4x^2 - 3}{x-2} = x^2 + 6x + 12 + \frac{21}{x-2}$ ;  $x^3 + 4x^2 - 3 = (x-2)(x^2 + 6x + 12) + 21$

5)  $k = -\frac{77}{27}$

6)  $m = 8, n = -10$

7) a)  $(x+1)(x-3)(x-2)$     b)  $(x+2)(3x+1)(x-4)$     c)  $-4(x+1)(x+2)(x-2)$     d)  $(x-4)(x^2 + 4x + 16)$

8) a)  $(-4, 0)$  and  $(4, 0)$     b)  $\left(\frac{1-\sqrt{105}}{4}, 0\right)$  and  $\left(\frac{1+\sqrt{105}}{4}, 0\right)$

9) a)  $x = -1, 1, \frac{1}{2}$     b)  $x = -1, -2, -3$     c)  $x = 1, -2, 2$     d)  $x = -1, -\frac{5}{3}, 2$

10) a)  $x \in (-\infty, -1) \cup (0.5, 1)$     b)  $x \in (-3, -2) \cup (-1, \infty)$

11)  $x \in \left(-\frac{1}{2}, \infty\right)$

12)  $x \in (-\infty, -4] \cup \left[\frac{1}{3}, \frac{3}{2}\right]$

13)  $P(x) = k(x-3)^2(x^2 - 4x + 2)$

14)  $f(x) = -\frac{1}{4}(x+2)(x+3)(x+5)$ , y-int is  $\left(0, -\frac{15}{2}\right)$

15) a)  $P(x) = -2x(x-1)(x+2)(2x+7)$     b)  $P(x) = -3(x+2)^2(x-1)$