

W1 - 3.4 Solve Rational Equations and Inequalities

MHF4U

Jensen

SOLUTIONS

1) Solve each equation algebraically.

a) $\frac{6}{2x-1} = 5$

$(2x-1) \left(\frac{6}{2x-1} \right) = 5(2x-1)$

$6 = 5(2x-1)$

$6 = 10x - 5$

$11 = 10x$

$x = \frac{11}{10}$

b) $\frac{6}{x} = x - 5$

$x \left(\frac{6}{x} \right) = x(x-5)$

$6 = x(x-5)$

$6 = x^2 - 5x$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x-6 = 0$

$x_1 = 6$

$x+1 = 0$

$x_2 = -1$

c) $1 = \frac{5}{3x^2-8x+2}$

$(3x^2-8x+2)(1) = \frac{5}{3x^2-8x+2} (3x^2-8x+2)$

$3x^2 - 8x + 2 = 5$

$3x^2 - 8x - 3 = 0$

$3x^2 - 9x + 1x - 3 = 0$

$3x(x-3) + 1(x-3) = 0$

$(x-3)(3x+1) = 0$

$x-3 = 0$

$3x+1 = 0$

$x_1 = 3$

$x_2 = -\frac{1}{3}$

d) $\frac{x+5}{x-1} = \frac{x+1}{x-3}$

$(x-3)(x+5) = (x+1)(x-1)$

$x^2 + 2x - 15 = x^2 - 1$

$2x = 14$

$x = 7$

$$e) \frac{3}{x+5} + \frac{4}{x} = 0$$

$$\frac{3}{x+5} = -\frac{4}{x}$$

$$3x = -4(x+5)$$

$$3x = -4x - 20$$

$$7x = -20$$

$$x = -\frac{20}{7}$$

$$f) 2x = 5 - \frac{3}{x}$$

$$x(2x) = x\left(5 - \frac{3}{x}\right)$$

$$2x^2 = 5x - 3$$

$$2x^2 - 5x + 3 = 0$$

$$2x^2 - 2x - 3x + 3 = 0$$

$$2x(x-1) - 3(x-1) = 0$$

$$(x-1)(2x-3) = 0$$

$$x-1 = 0$$

$$2x-3 = 0$$

$$x_1 = 1$$

$$x_2 = \frac{3}{2}$$

$$g) \frac{2}{1-x} + \frac{3}{x+1} = \frac{1}{x}$$

$$\frac{2(x+1) + 3(1-x)}{(1-x)(x+1)} = \frac{1}{x}$$

$$\frac{2x+2+3-3x}{1-x^2} = \frac{1}{x}$$

$$\frac{-1x+5}{1-x^2} = \frac{1}{x}$$

$$x(-1x+5) = 1(1-x^2)$$

$$-1x^2 + 5x = 1 - 1x^2$$

$$5x = 1$$

$$x = \frac{1}{5}$$

$$h) \frac{3}{x-1} + 5 + \frac{2}{x} = 0$$

$$x(x-1) \left[\frac{3}{x-1} + 5 + \frac{2}{x} \right] = 0 \quad (x)(x-1)$$

$$3x + 5x(x-1) + 2(x-1) = 0$$

$$3x + 5x^2 - 5x + 2x - 2 = 0$$

$$5x^2 = 2$$

$$x^2 = \frac{2}{5}$$

$$x = \pm \sqrt{\frac{2}{5}}$$

2) Solve each inequality without using technology.

a) $\frac{4}{2x-3} < \frac{1}{x+4}$

(2x+4) $\frac{4}{2x-3} - \frac{1}{x+4} < 0$
 (2x-3) $\frac{4(x+4) - (2x-3)}{(2x-3)(x+4)} < 0$

$\frac{4x+16-2x+3}{(2x-3)(x+4)} < 0$

$\frac{2x+19}{(2x-3)(x+4)} < 0$

	$-\infty$	-9.5	-4	1.5	∞
$2x+19$	-	+	+	+	
$2x-3$	-	-	-	+	
$x+4$	-	-	+	+	
overall	-	+	-	+	

zero: $x = -9.5$

restrictions: $x \neq -4, 1.5$

Solution: $x < -9.5$ or $-4 < x < 1.5$
 $x \in (-\infty, -9.5) \cup (-4, 1.5)$

b) $\frac{2x+3}{x-3} \geq \frac{6x-5}{3x+1}$

(3x+1) $\frac{2x+3}{x-3} - \frac{6x-5}{3x+1} \geq 0$
 (3x+1)(x-3) $\frac{(3x+1)(2x+3) - (6x-5)(x-3)}{(3x+1)(x-3)} \geq 0$

$\frac{6x^2+11x+3 - (6x^2-23x+15)}{(3x+1)(x-3)} \geq 0$

$\frac{34x-12}{(3x+1)(x-3)} \geq 0$

$\frac{2(17x-6)}{(3x+1)(x-3)} \geq 0$

	$-\infty$	$-\frac{6}{17}$	$\frac{6}{17}$	3	∞
$17x-6$	-	+	+	+	
$3x+1$	-	+	+	+	
$x-3$	-	-	-	+	
overall	-	+	-	+	

zero: $\frac{6}{17}$

restrictions: $-\frac{1}{3}, 3$

Solution: $-\frac{1}{3} < x \leq \frac{6}{17}$ or $x > 3$
 $x \in (-\frac{1}{3}, \frac{6}{17}] \cup (3, \infty)$

c) $\frac{(x-3)(2x-1)}{(x+4)(x-5)} > 0$

zeros: $\frac{1}{2}, 3$

restrictions: -4, 5

	$-\infty$	-4	0.5	3	5	∞
$x-3$	-	-	-	+	+	
$2x-1$	-	-	+	+	+	
$x+4$	-	+	+	+	+	
$x-5$	-	-	-	-	+	
overall	+	-	+	-	+	

Solution: $x < -4$ or $0.5 < x < 3$ or $x > 5$
 $x \in (-\infty, -4) \cup (0.5, 3) \cup (5, \infty)$

d) $\frac{2x^2+5x-3}{x^2+5x+4} \leq 0$



$\frac{(2x-1)(x+3)}{(x+4)(x+1)} \leq 0$

zeros: $-3, \frac{1}{2}$

restrictions: $-4, -1$

	$-\infty$	-4	-3	-1	$\frac{1}{2}$	∞
$2x-1$	-	-	-	-	+	
$x+3$	-	-	+	+	+	
$x+4$	-	+	+	+	+	
$x+1$	-	-	-	+	+	
overall	+	-	+	-	+	

Solution: $-4 < x \leq -3$ or $-1 < x \leq \frac{1}{2}$
 $x \in (-4, -3] \cup (-1, \frac{1}{2}]$

$$e) \frac{4}{x-3} < 1$$

$$\frac{4}{x-3} - 1 < 0$$

$$\frac{4}{x-3} - \frac{x-3}{x-3} < 0$$

$$\frac{4-(x-3)}{x-3} < 0$$

$$\frac{4-x+3}{x-3} < 0$$

$$\frac{7-x}{x-3} < 0$$

	$-\infty$	3	7	∞
$7-x$	+	+	-	
$x-3$	-	+	+	
overall	-	+	-	

zero: $x=7$

restriction: $x \neq 3$

Solution: $x < 3$ or $x > 7$

$$x \in (-\infty, 3) \cup (7, \infty)$$

$$-1 \frac{x}{x+3} > \frac{x}{x-1}$$

$$\frac{x}{x+3} - \frac{x}{x-1} > 0$$

$$\frac{x(x-1) - x(x+3)}{(x+3)(x-1)} > 0$$

$$\frac{x^2 - x - x^2 - 3x}{(x+3)(x-1)} > 0$$

$$\frac{-4x}{(x+3)(x-1)} > 0$$

	$-\infty$	-3	0	1	∞
$-4x$	+	+	-	-	
$x+3$	-	+	+	+	
$x-1$	-	-	-	+	
overall	+	-	+	-	

zero: $x=0$

restrictions: $x \neq -3, 1$

Solution: $x < -3$ or $0 < x < 1$

$$x \in (-\infty, -3) \cup (0, 1)$$

$$f) \frac{2x^2+5x-3}{x^2+8x+16} < 0$$

$$\left(\frac{3}{1}\right) = \frac{6}{2} \times \frac{-1}{3}$$

$$\frac{(2x-1)(x+3)}{(x+4)^2} < 0$$

$$(x+4)^2$$

zeros: $x = -3, \frac{1}{2}$

restriction: $x \neq -4$

	$-\infty$	-4	-3	$\frac{1}{2}$	∞
$2x-1$	-	-	-	+	
$x+3$	-	-	+	+	
$(x+4)^2$	+	+	+	+	
overall	+	+	-	+	

Solution: $-3 < x < \frac{1}{2}$

$$x \in (-3, \frac{1}{2})$$

$$h) \frac{2x+3}{x} > \frac{x+1}{x}$$

$$\frac{2x+3}{x} - \frac{x+1}{x} > 0$$

$$\frac{2x+3-(x+1)}{x} > 0$$

$$\frac{2x+3-x-1}{x} > 0$$

$$\frac{x+2}{x} > 0$$

	$-\infty$	-2	0	∞
$x+2$	-	+	+	
x	-	-	+	
overall	+	-	+	

zero: $x = -2$

restriction: $x \neq 0$

Solution: $x < -2$ or $x > 0$

$$x \in (-\infty, -2) \cup (0, \infty)$$

Answer Key

1) a) $x = 1.1$ b) $x = 6$ or $x = -1$ c) $x = 3$ or $x = -\frac{1}{3}$ d) $x = 7$ e) $x = -\frac{20}{7}$ f) $x = 1$ or $x = \frac{3}{2}$
g) $x = \frac{1}{5}$ h) $x = \pm\sqrt{0.4}$

2) a) $x < -\frac{19}{2}$ or $-4 < x < \frac{3}{2}$ b) $-\frac{1}{3} < x \leq \frac{6}{17}$ or $x > 3$ c) $x < -4$ or $\frac{1}{2} < x < 3$ or $x > 5$

d) $-4 < x \leq -3$ or $-1 < x \leq \frac{1}{2}$ e) $x < 3$ or $x > 7$ f) $-3 < x < \frac{1}{2}$

g) $x < -3$ or $0 < x < 1$ h) $x < -2$ or $x > 0$