W1 – Increasing / Decreasing Unit 2

MCV4U

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SOLUTIONS

1) Use critical numbers and the first derivative test to determine when the function is increasing or decreasing.

a) $f(x) = x^3 + 3x^2 + 1$

$f'(x) = 3x^2 + 6x$

$0 = 3x(x+2)$

$x_1 = 0 \quad x_2 = -2$

$-\infty$	-2	0	∞
Test value	-3	-1	1
$f'(x)$	+	−	+
$f(x)$	increasing	decreasing	increasing

increasing: $x < -2$, $x > 0$

decreasing: $-2 < x < 0$

b) $f(x) = x^5 - 5x^4 + 100$

$f'(x) = 5x^4 - 20x^3$

$0 = 5x^3(x-4)$

$x_1 = 0 \quad x_2 = 4$

$-\infty$	0	4	∞
Test value	-1	1	5
$f'(x)$	+	−	+
$f(x)$	inc.	dec.	inc.

increasing: $x < 0$, $x > 4$

decreasing: $0 < x < 4$

c) $f(x) = 3x^4 + 4x^3 - 12x^2$

$f'(x) = 12x^3 + 12x^2 - 24x$

$0 = 12x(x^2 + x - 2)$

$0 = 12x(x+2)(x-1)$

$x_1 = 0 \quad x_2 = -2 \quad x_3 = 1$

$-\infty$	-2	0	1	∞
Test	-3	-1	0.5	2
$f'(x)$	−	+	−	+
$f(x)$	dec.	inc.	dec.	inc.

increasing: $-2 < x < 0$, $x > 1$

decreasing: $x < -2$, $0 < x < 1$

d) $f(x) = (2x-1)^2(x^2-9)$

$f'(x) = 2(2x-1)(2)(x^2-9) + 2x(2x-1)^2$

$0 = 2(2x-1)[2(x^2-9) + x(2x-1)]$

$0 = 2(2x-1)(4x^2 - x - 18)$

$0 = 2(2x-1)[4x^2 - 9x + 8x - 18]$

$0 = 2(2x-1)[x(4x-9) + 2(4x-9)]$

$0 = 2(2x-1)(4x-9)(x+2)$

$x_1 = \frac{1}{2} \quad x_2 = \frac{9}{4} = 2.25 \quad x_3 = -2$

	$-\infty$ to -2	-2 to 0.5	0.5 to 2.25	2.25 to ∞
Test	-3	0	1	3
$f'(x)$	$-$	$+$	$-$	$+$
$f(x)$	dec.	inc.	dec	inc.

increasing: $-2 < x < 0.5,\ x > 2.25$

decreasing: $x < -2,\ 0.5 < x < 2.25$

2) Suppose that $f(x)$ is a differentiable function with the given derivative. Determine the values of x for which $f(x)$ is increasing and decreasing.

a) $f'(x) = (x-1)(x+2)(x+3)$

$0 = (x-1)(x+2)(x+3)$

$x_1 = 1 \quad x_2 = -2 \quad x_3 = -3$

	$-\infty$ to -3	-3 to -2	-2 to 1	1 to ∞
test value	-4	-2.5	0	2
$f'(x)$	$-$	$+$	$-$	$+$
$f(x)$	dec.	inc.	dec.	inc.

Increasing: $-3 < x < -2,\ x > 1$

decreasing: $x < -3,\ -2 < x < 1$

b) $f'(x) = x^2 + 2x - 4$

$0 = x^2 + 2x - 4$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{20}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{5}}{2}$

$x = -1 \pm \sqrt{5}$

$x_1 \simeq 1.24 \quad x_2 \simeq -3.24$

	$-\infty$ to $-1-\sqrt{5}$	$-1-\sqrt{5}$ to $-1+\sqrt{5}$	$-1+\sqrt{5}$ to ∞
Test Value	-4	0	2
$f'(x)$	$+$	$-$	$+$
$f(x)$	inc.	dec.	inc.

increasing: $x < -1-\sqrt{5},\ x > -1+\sqrt{5}$

decreasing: $-1-\sqrt{5} < x < -1+\sqrt{5}$

c) $f'(x) = x^3 + 3x^2 - 4x - 12$

$0 = x^2(x+3) - 4(x+3)$

$0 = (x+3)(x^2-4)$

$0 = (x+3)(x-2)(x+2)$

$x_1 = -3 \quad x_2 = 2 \quad x_3 = -2$

$-\infty \quad -3 \quad -2 \quad 2 \quad \infty$

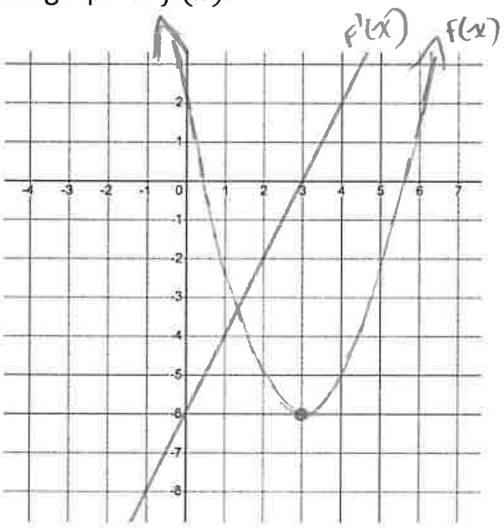
Test value	-4	-2.5	0	3
$f'(x)$	−	+	−	+
$f(x)$	dec.	inc	dec.	inc.

Increasing: $-3 < x < -2,\ x > 2$

decreasing: $x < -3,\ -2 < x < 2$

3) Given each graph of $f'(x)$, state the intervals of increase and decrease for the function $f(x)$. Then sketch a possible graph of $f(x)$.

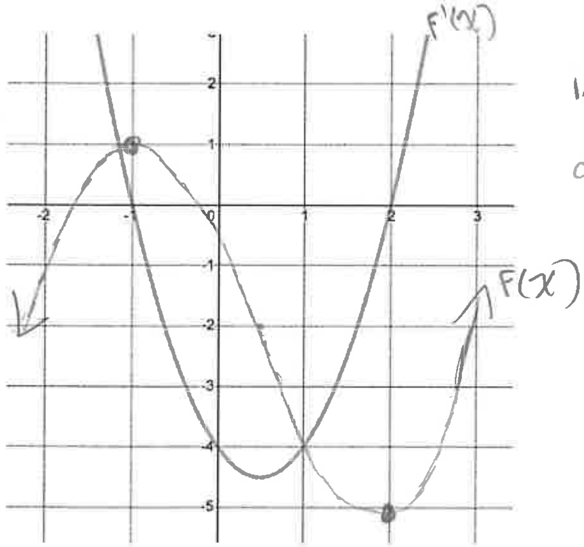
a)



increasing: $x > 3$

decreasing: $x < 3$

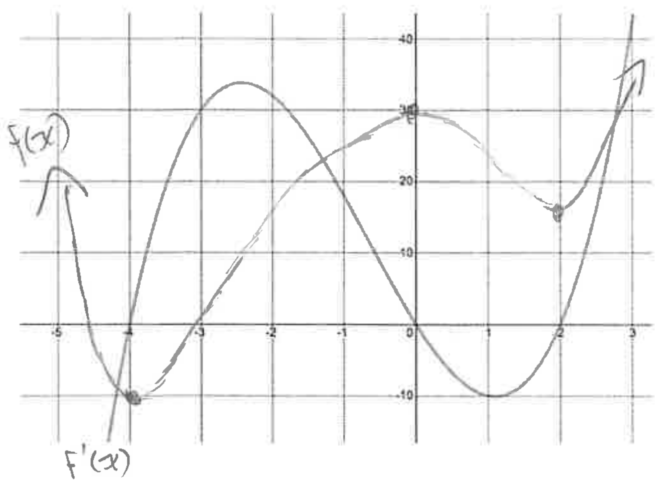
b)



Increasing: $x < -1,\ x > 2$

decreasing: $-1 < x < 2$

c)



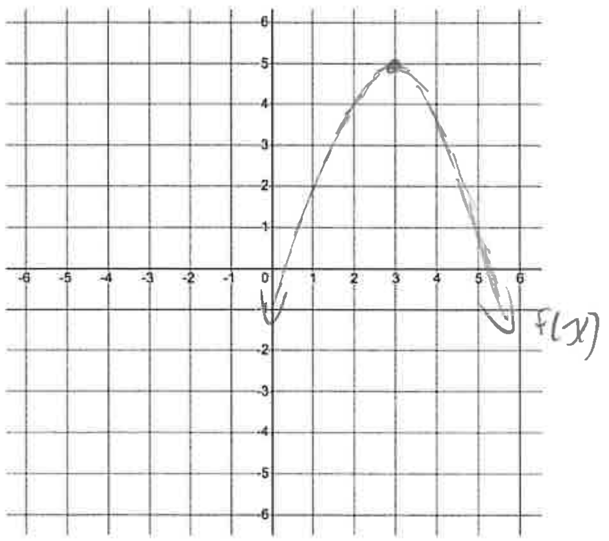
increasing: $-4<x<0$, $x>2$

decreasing: $x<-4$, $0<x<2$

4) Sketch a continuous graph of $f(x)$ given each set of conditions.

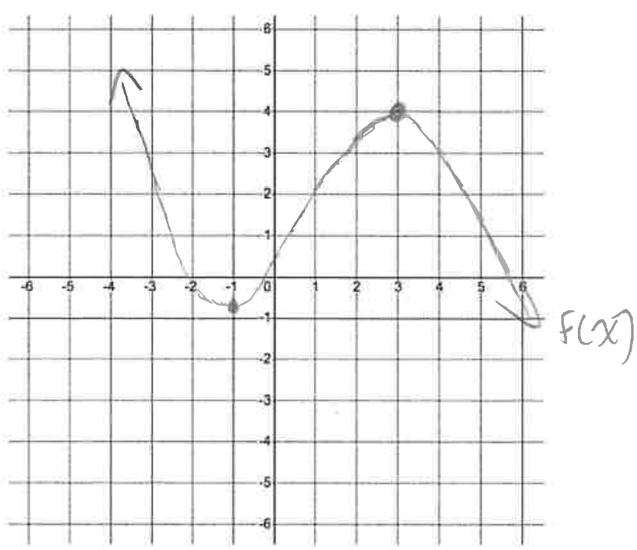
increasing ... decreasing

a) $f'(x) > 0$ when $x < 3$, $f'(x) < 0$ when $x > 3$, $f(3) = 5$

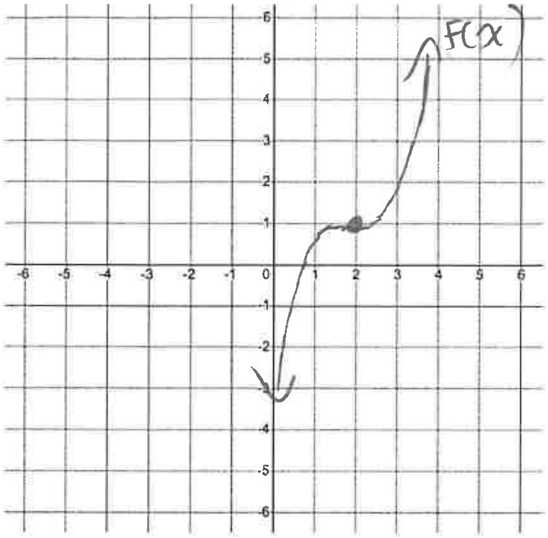


increasing ... decreasing

b) $f'(x) > 0$ when $-1 < x < 3$, $f'(x) < 0$ when $x < -1$ and when $x > 3$, $f(-1) = -\frac{20}{27}$, $f(3) = 4$



c) $f'(x) > 0$ when $x \neq 2$, $f(2) = 1$



Answers:

1)a) increasing: $x < -2, x > 0$
decreasing: $-2 < x < 0$

b) increasing: $x < 0, x > 4$
decreasing: $0 < x < 4$

c) increasing: $-2 < x < 0, x > 1$
decreasing: $x < -2, 0 < x < 1$

d) increasing: $-2 < x < 0.5, x > 2.25$
decreasing: $x < -2, 0.5 < x < 2.25$

2)a) increasing: $-3 < x < -2, x > 1$
decreasing: $x < -3, -2 < x < 1$

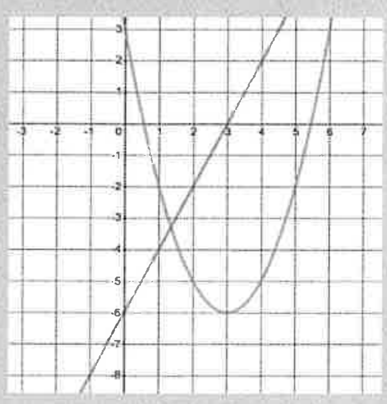
b) increasing: $x < -1 - \sqrt{5}, x > -1 + \sqrt{5}$
decreasing: $-1 - \sqrt{5} < x < -1 + \sqrt{5}$

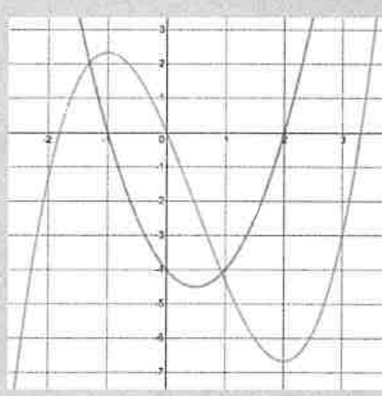
c) increasing: $-3 < x < -2, x > 2$
decreasing: $x < -3, -2 < x < 2$

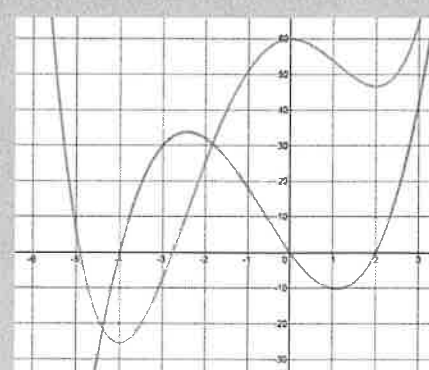
3a) increasing: $x > 3$
decreasing: $x < 3$

b) increasing: $x < -1, x > 2$
decreasing: $-1 < x < 2$

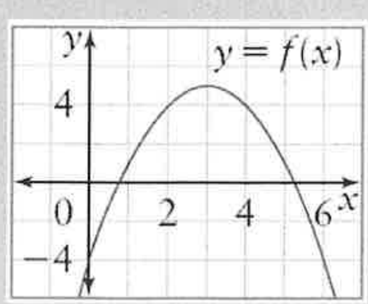
c) increasing: $-4 < x < 0, x > 2$
decreasing: $x < -4, 0 < x < 2$



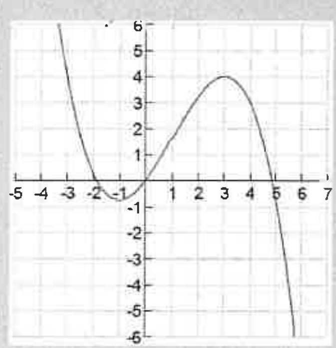




4)a)



b)



c)