W1 – Equations of Lines in Two-Space Unit 6

MCV4U

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SOLUTIONS

1) Write a vector equation for a line given each direction vector $\vec{m}$ and point P_0.

a) $\vec{m} = [2, -7]$, $P_0(9,4)$

$[x,y] = [9,4] + t[2,-7]$

b) $\vec{m} = [-1, 5]$, $P_0(3,7)$

$[x,y] = [3,7] + t[-1,5]$

2) Write the vector equation of the line that passes through each pair of points.

a) $A(2,3)$, $B(7,1)$

$\vec{m} = [7-2, 1-3]$

$\vec{m} = [5, -2]$

$[x,y] = [2,3] + t[5,-2]$

b) $A(5,-1)$, $B(-2,7)$

$\vec{m} = [-2-5, 7-(-1)]$

$\vec{m} = [-7, 8]$

$[x,y] = [5,-1] + t[-7,8]$

3) Write the parametric equations for each vector equation.

a) $[x, y] = [7,3] + t[2,5]$

$l: \begin{cases} x = 7+2t \\ y = 3+5t \end{cases}$

b) $[x, y] = [-1,4] + t[8,-9]$

$l: \begin{cases} x = -1+8t \\ y = 4-9t \end{cases}$

4) Determine if each point P is on the line $[x, y] = [3,1] + t[-2,5]$

$l: \begin{cases} x = 3-2t \\ y = 1+5t \end{cases}$

a) P(12, 36)

$12 = 3-2t$ $\quad 36 = 1+5t$

$9 = -2t$ $\quad 35 = 5t$

$t = -\frac{9}{2}$ $\quad t = 7$

∴ P(12,36) is NOT on the line

b) P(−7, 26)

$-7 = 3-2t$ $\quad 26 = 1+5t$

$-10 = -2t$ $\quad 25 = 5t$

$t = 5$ $\quad t = 5$

∴ P(−7,26) IS on the line.

c) P(1, 6)

$1 = 3-2t$ $\quad 6 = 1+5t$

$-2 = -2t$ $\quad 5 = 5t$

$t = 1$ $\quad t = 1$

∴ P(1,6) IS on the line

d) P(−3, 15)

$-3 = 3-2t$ $\quad 15 = 1+5t$

$-6 = -2t$ $\quad 14 = 5t$

$t = 3$ $\quad t = \frac{14}{5}$

∴ P(−3,15) is NOT on the line.

5) Write a vector equation for each line, given the parametric equations.

a) $\ell: \begin{cases} x = 2 - 4t \\ y = 1 - 5t \end{cases}$

$[x, y] = [2, 1] + t[-4, -5]$

b) $\ell: \begin{cases} x = 8 - 9t \\ y = 3 \end{cases}$

$[x, y] = [8, 3] + t[-9, 0]$

6) Given each set of parametric equations, write the scalar equation.

a) $\ell: \begin{cases} x = 3 + 4t \\ y = -1 - 5t \end{cases}$

$x = 3 + 4t \qquad y = -1 - 5t$

$\frac{x-3}{4} = t \qquad \frac{y+1}{-5} = t$

$\frac{x-3}{4} = \frac{y+1}{-5}$

$-5x + 15 = 4y + 4$

$0 = 5x + 4y - 11$

b) $\ell: \begin{cases} x = 2 + 8t \\ y = -5 + 7t \end{cases}$

$x = 2 + 8t \qquad y = -5 + 7t$

$\frac{x-2}{8} = t \qquad \frac{y+5}{7} = t$

$\frac{x-2}{8} = \frac{y+5}{7}$

$7x - 14 = 8y + 40$

$7x - 8y - 54 = 0$

7) Write the scalar equation of each line given the normal vector $\vec{n}$ and point P_0.

a) $\vec{n} = [4,1]$, $P_0(3, -5)$

$Ax + By + C = 0$

$4(3) + 1(-5) + C = 0$

$C = -7$

$4x + y - 7 = 0$

b) $\vec{n} = [1, -3]$, $P_0(4,3)$

$Ax + By + C = 0$

$1(4) + (-3)(3) + C = 0$

$C = 5$

$x - 3y + 5 = 0$

8) Determine the vector equation of each line.

a) parallel to the *y*-axis and through $P_0(2, -7)$

$\vec{m} = [0, 1]$

$[x, y] = [2, -7] + t[0, 1]$

b) parallel to $[x, y] = [3, -1] + t[3, 4]$ and through $P_0(-2, 4)$

$\vec{m} = [3, 4]$

$[x, y] = [-2, 4] + t[3, 4]$

c) perpendicular to $[x, y] = [2, -1] + t[5, 3]$ with *x*-intercept 3

$\vec{m} = [-3, 5]$

$\vec{r}_0 = [3, 0]$

$[x, y] = [3, 0] + t[-3, 5]$

9) Given each scalar equation, write a vector equation and the parametric equations.

a) $x + 2y = 6$

x-int: $x = 6$ $(6,0)$

y-int: $2y = 6$; $y = 3$ $(0,3)$

$\vec{m} = [-6,3] = 3[-2,1]$

$[x,y] = [6,0] + t[-2,1]$

$\ell: \begin{cases} x = 6 - 2t \\ y = t \end{cases}$

b) $5x - 2y = 13$

$\vec{n} = [5,-2]$

$\vec{m} = [2,5]$

Point: $5(1) - 2y = 13$

$-8 = 2y$

$y = -4$ $(1,-4)$

$[x,y] = [1,-4] + t[2,5]$

$\ell: \begin{cases} x = 1 + 2t \\ y = -4 + 5t \end{cases}$

10) In each case, determine if ℓ_1 and ℓ_2 are parallel, perpendicular, or neither. Explain.

a) $\ell_1: 4x - 6y = 9$

$\ell_2: [x,y] = [6,3] + t[3,2]$

$\vec{n}_1 = [4,-6]$

$\vec{m}_1 = [6,4] = 2[3,2]$

$\vec{m}_2 = [3,2]$

$\therefore \vec{m}_1 = 2\vec{m}_2$

$\therefore \ell_1$ and ℓ_2 are parallel

b) $\ell_1: x + 9y = 2$

$\ell_2: \begin{cases} x = t \\ y = 15 + 9t \end{cases}$

$\vec{n}_1 = [1,9]$

$\vec{m}_1 = [-9,1]$

$\vec{m}_2 = [1,9]$

$\vec{m}_1 \neq k\vec{m}_2$

$\therefore$ not parallel

$\vec{m}_1 \cdot \vec{m}_2 = [-9,1] \cdot [1,9]$

$= -9(1) + 1(9)$

$= 0$

$\therefore \ell_1$ and ℓ_2 are perpendicular.

Answers:

1)a) $[x, y] = [9, 4] + t[2, -7]$ **b)** $[x, y] = [3, 7] + t[-1, 5]$

2)a) $[x,y] = [2,3] + t[5,-2]$ **b)** $[x,y] = [5,-1] + t[-7,8]$

3)a) $\ell: \begin{cases} x = 7 + 2t \\ y = 3 + 5t \end{cases}$ **b)** $\ell: \begin{cases} x = -1 + 8t \\ y = 4 - 9t \end{cases}$

4)a) No **b)** Yes **c)** Yes **d)** No

5)a) $[x, y] = [2, 1] + t[-4, -5]$ **b)** $[x, y] = [8, 3] + t[-9, 0]$

6)a) $5x + 4y - 11 = 0$ **b)** $-7x + 8y + 54 = 0$

7)a) $4x + y - 7 = 0$ **b)** $x - 3y + 5 = 0$

8)a) $[x, y] = [2, -7] + t[0, 1]$ **b)** $[x, y] = [-2, 4] + t[3, 4]$ **c)** $[x, y] = [3, 0] + t[-3, 5]$

9)a) $[x,y] = [2,2] + t[-2,1]; x = 2 - 2t, y = 2 + t$ **b)** $[x,y] = [3,1] + t[2,5]; x = 3 + 2t, y = 1 + 5t$

10)a) parallel **b)** perpendicular