

W2 – Equations of Lines in Three-Space

Unit 6

MCV4U

Jensen

1) Write a vector equation for a line given each direction vector \vec{m} and point P_0 .

a) $\vec{m} = [2, -3, 5], P_0(8, -11, 2)$

b) $\vec{m} = [-3, 4, 7], P_0(7, -1, 4)$

2) Write the vector equation of the line that passes through each pair of points.

a) $A(4, -3, 1), B(2, -3, 7)$

b) $A(5, 6, -1), B(0, -3, 2)$

3) Write the parametric equations for each vector equation.

a) $[x, y, z] = [0, 4, -7] + t[3, -4, 1]$

b) $[x, y, z] = [7, 5, -4] + t[-2, 1, 3]$

4) Write a vector equation for each line, given the parametric equations.

a) $\ell: \begin{cases} x = 4 + 3t \\ y = 7 - 2t \\ z = 1 + t \end{cases}$

b) $\ell: \begin{cases} x = 4t \\ y = 5 - 9t \\ z = -3 \end{cases}$

5) Write a vector equation and the parametric equations of a line going through the points $A(7, 8, -3)$ and $B(-2, 3, 5)$.

6) Determine the vector equation of a line through $P_0(-1, -7, 7)$ and perpendicular to $[x, y, z] = [1, -7, 3] + t[4, -1, 2]$.

7) Determine which points are on the line $[x, y, z] = [3, 1, -4] + t[2, 0, 5]$.

a) $(-5, 1, -24)$

b) $(19, 1, 37)$

8) State where possible vector, parametric, and symmetric equations for each of the following lines:

a) the line passing through the point $P(-1, 2, 1)$ with direction vector $(3, -2, 1)$

b) The line passing through the points $(-1, 1, 0)$ and $B(-1, 2, 1)$

c) The line passing through the point $Q(1,2,4)$ and parallel to the z -axis

9) Determine the parametric equations of the line whose direction vector is perpendicular to the direction vectors of the two lines $\frac{x}{-4} = \frac{y+10}{-7} = \frac{z+2}{3}$ and $\frac{x-5}{3} = \frac{y-5}{2} = \frac{z+5}{4}$ and passes through the point $(2, -5, 0)$.

Answers

1)a) $[x, y, z] = [8, -11, 2] + t[2, -3, 5]$ b) $[x, y, z] = [7, -1, 4] + t[-3, 4, 7]$

2)a) $[x, y, z] = [4, -3, 1] + t[-2, 0, 6]$ b) $[x, y, z] = [5, 6, -1] + t[-5, -9, 3]$

3) $\ell: \begin{cases} x = 3t \\ y = 4 - 4t \\ z = -7 + t \end{cases}$ b) $\ell: \begin{cases} x = 7 - 2t \\ y = 5 + t \\ z = -4 + 3t \end{cases}$

4)a) $[x, y, z] = [4, 7, 1] + t[3, -2, 1]$ b) $[x, y, z] = [0, 5, -3] + t[4, -9, 0]$

5) $[x, y, z] = [7, 8, -3] + t[-9, -5, 8]$; $\ell: \begin{cases} x = 7 - 9t \\ y = 8 - 5t \\ z = -3 + 8t \end{cases}$

6) $[x, y, z] = [-1, -7, 7] + t[0, 2, 1]$

7)a) Yes b) No

8)a) $[x, y, z] = [-1, 2, 1] + t[3, -2, 1]$; $\ell: \begin{cases} x = -1 + 3t \\ y = 2 - 2t \\ z = 1 + t \end{cases}$; $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z-1}{1}$

b) $[x, y, z] = [-1, 1, 0] + t[0, 1, 1]$; $\ell: \begin{cases} x = -1 \\ y = 1 + t \\ z = t \end{cases}$; $\frac{y-1}{1} = \frac{z}{1}$, $x = -1$

c) $[x, y, z] = [1, 2, 4] + t[0, 0, 1]$; $\ell: \begin{cases} x = 1 \\ y = 2 \\ z = 4 + t \end{cases}$; no symmetric equation for this line since two of the direction vector coordinates are zero.

9) $\ell: \begin{cases} x = 2 - 34t \\ y = -5 + 25t \\ z = 13t \end{cases}$