W2 – Equations of Lines in Three-Space Unit 6

MCV4U

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SOLUTIONS

1) Write a vector equation for a line given each direction vector $\vec{m}$ and point P_0.

a) $\vec{m} = [2,-3,5]$, $P_0(8,-11,2)$

$[x,y,z] = [8,-11,2] + t[2,-3,5]$

b) $\vec{m} = [-3,4,7]$, $P_0(7,-1,4)$

$[x,y,z] = [7,-1,4] + t[-3,4,7]$

2) Write the vector equation of the line that passes through each pair of points.

a) $A(4,-3,1)$, $B(2,-3,7)$

$\vec{m} = [2-4,-3-(-3),7-1]$

$\vec{m} = [-2,0,6] = -2[1,0,-3]$

$[x,y,z] = [4,-3,1] + t[1,0,-3]$

b) $A(5,6,-1)$, $B(0,-3,2)$

$\vec{m} = [0-5,-3-6,2-(-1)]$

$\vec{m} = [-5,-9,3]$

$[x,y,z] = [5,6,-1] + t[-5,-9,3]$

3) Write the parametric equations for each vector equation.

a) $[x,y,z] = [0,4,-7] + t[3,-4,1]$

$l: \begin{cases} x = 3t \\ y = 4-4t \\ z = -7+t \end{cases}$

b) $[x,y,z] = [7,5,-4] + t[-2,1,3]$

$l: \begin{cases} x = 7-2t \\ y = 5+t \\ z = -4+3t \end{cases}$

4) Write a vector equation for each line, given the parametric equations.

a) $\ell: \begin{cases} x = 4+3t \\ y = 7-2t \\ z = 1+t \end{cases}$

$[x,y,z] = [4,7,1] + t[3,-2,1]$

b) $\ell: \begin{cases} x = 4t \\ y = 5-9t \\ z = -3 \end{cases}$

$[x,y,z] = [0,5,-3] + t[4,-9,0]$

5) Write a vector equation and the parametric equations of a line going through the points A(7, 8, –3) and B(–2, 3, 5).

$\vec{m} = [7-(-2),8-3,-3-5]$

$\vec{m} = [9,5,-8]$

$[x,y,z] = [7,8,-3] + t[9,5,-8]$

$l: \begin{cases} x = 7+9t \\ y = 8+5t \\ z = -3-8t \end{cases}$

6) Determine the vector equation of a line through $P_0(-1, -7, 7)$ and perpendicular to $[x, y, z] = [1, -7, 3] + t[4, -1, 2]$.

$\vec{n} = [4, -1, 2]$

$\vec{m} = [0, 2, 1]$

$[x, y, z] = [-1, -7, 7] + t[0, 2, 1]$

7) Determine which points are on the line $[x, y, z] = [3, 1, -4] + t[2, 0, 5]$.

a) (–5, 1, –24)

$x = 3 + 2t$ $\quad y = 1$ $\quad z = -4 + 5t$

$-5 = 3 + 2t$ $\quad 1 = 1$ $\quad -24 = -4 + 5t$

$-8 = 2t$ $\quad -20 = 5t$

$t = -4$ $\quad t = -4$

ON line.

b) (19, 1, 37)

$19 = 3 + 2t$ $\quad 1 = 1$ $\quad 37 = -4 + 5t$

$16 = 2t$ $\quad 41 = 5t$

$t = 8$ $\quad t = \frac{41}{5}$

NOT on line.

8) State where possible vector, parametric, and symmetric equations for each of the following lines:

a) the line passing through the point $P(-1,2,1)$ with direction vector $(3, -2,1)$

$[x, y, z] = [-1, 2, 1] + t[3, -2, 1]$

$\frac{x+1}{3} = \frac{y-2}{-2} = z - 1$

$l: \begin{cases} x = -1 + 3t \\ y = 2 - 2t \\ z = 1 + t \end{cases}$

b) The line passing through the points $A(-1,1,0)$ and $B(-1,2,1)$

$\vec{m} = [-1 - (-1), 2 - 1, 1 - 0]$

$\vec{m} = [0, 1, 1]$

$[x, y, z] = [-1, 1, 0] + t[0, 1, 1]$

$l: \begin{cases} x = -1 \\ y = 1 + t \\ z = t \end{cases}$ $\quad \frac{y-1}{1} = \frac{z}{1}$; $x = -1$

c) The line passing through the point $Q(1,2,4)$ and parallel to the z-axis

$\vec{m} = [0,0,1]$

$[x,y,z] = [1,2,4] + t[0,0,1]$

$\ell: \begin{cases} x = 1 \\ y = 2 \\ z = 4+t \end{cases}$

No symmetric possible.

9) Determine the parametric equations of the line whose direction vector is perpendicular to the direction vectors of the two lines $\frac{x}{-4} = \frac{y+10}{-7} = \frac{z+2}{3}$ and $\frac{x-5}{3} = \frac{y-5}{2} = \frac{z+5}{4}$ and passes through the point $(2,-5,0)$.

l_1:

$\frac{x}{-4} = t \quad \frac{y+10}{-7} = t \quad \frac{z+2}{3} = t$

$x = -4t \quad y = -7t-10 \quad z = 3t-2$

$\vec{m}_1 = [-4,-7,3]$

$\vec{m}_1 \times \vec{m}_2 = [-7(4)-3(2), 3(3)-(-4)(4), -4(2)-(-7)(3)]$

$\begin{matrix} -7 & 2 \\ 3 & 4 \\ -4 & 3 \\ -7 & 2 \end{matrix} = [-34, 25, 13]$

l_2: $\frac{x-5}{3} = t \quad \frac{y-5}{2} = t \quad \frac{z+5}{4} = t$

$x = 3t+5 \quad y = 2t+5 \quad z = 4t-5$

$\vec{m}_2 = [3,2,4]$

$[x,y,z] = [2,-5,0] + t[-34,25,13]$

$l_3: \begin{cases} x = 2-34t \\ y = -5+25t \\ z = 13t \end{cases}$

Answers

1)a) $[x, y, z] = [8, -11, 2] + t[2, -3, 5]$ **b)** $[x, y, z] = [7, -1, 4] + t[-3, 4, 7]$

2)a) $[x, y, z] = [4, -3, 1] + t[-2, 0, 6]$ **b)** $[x, y, z] = [5, 6, -1] + t[-5, -9, 3]$

3) $\ell: \begin{cases} x = 3t \\ y = 4 - 4t \\ z = -7 + t \end{cases}$ **b)** $\ell: \begin{cases} x = 7 - 2t \\ y = 5 + t \\ z = -4 + 3t \end{cases}$

4)a) $[x, y, z] = [4, 7, 1] + t[3, -2, 1]$ **b)** $[x, y, z] = [0, 5, -3] + t[4, -9, 0]$

5) $[x, y, z] = [7, 8, -3] + t[-9, -5, 8]$; $\ell: \begin{cases} x = 7 - 9t \\ y = 8 - 5t \\ z = -3 + 8t \end{cases}$

6) $[x, y, z] = [-1, -7, 7] + t[0, 2, 1]$

7)a) Yes **b)** No

8)a) $[x, y, z] = [-1,2,1] + t[3, -2, 1]$; $\ell: \begin{cases} x = -1 + 3t \\ y = 2 - 2t \\ z = 1 + t \end{cases}$; $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z-1}{1}$

b) $[x, y, z] = [-1,1,0] + t[0,1,1]$; $\ell: \begin{cases} x = -1 \\ y = 1 + t \\ z = t \end{cases}$; $\frac{y-1}{1} = \frac{z}{1}, x = -1$

c) $[x, y, z] = [1,2,4] + t[0,0,1]$; $\ell: \begin{cases} x = 1 \\ y = 2 \\ z = 4 + t \end{cases}$; no symmetric equation for this line since two of the direction vector coordinates are zero.

9) $\ell: \begin{cases} x = 2 - 34t \\ y = -5 + 25t \\ z = 13t \end{cases}$