W2 – Maxima and Minima Unit 2

MCV4U

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SOLUTIONS

1) Find the critical numbers for each function

a) $f(x) = -x^2 + 6x + 2$

$f'(x) = -2x+6$

$0 = -2x+6$

$x = 3$

b) $f(x) = x^3 - 2x^2 + 3x$

$f'(x) = 3x^2 - 4x + 3$

$0 = 3x^2 - 4x + 3$

$b^2 - 4ac = (-4)^2 - 4(3)(3)$

$b^2 - 4ac = -20$

$b^2 - 4ac < 0$ $\therefore$ no critical #'s

c) $g(x) = 2x^3 - 3x^2 - 12x + 5$

$g'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x_1 = -1 \quad x_2 = 2$

d) $y = x - \sqrt{x}$

$\frac{dy}{dx} = 1 - \frac{1}{2}x^{-1/2}$

$\frac{dy}{dx} = 1 - \frac{1}{2\sqrt{x}}$

$0 = 1 - \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1$

$1 = 2\sqrt{x}$

$(\frac{1}{2})^2 = x$

$x = \frac{1}{4}$

2) Determine the absolute extreme values of each function on the given interval.

a) $y = 3x^2 - 12x + 7, 0 \leq x \leq 4$

$\frac{dy}{dx} = 6x - 12$

$0 = 6x - 12$

$x = 2$ is a critical #

$y(0) = 3(0)^2 - 12(0) + 7$

$= 7$

$y(2) = 3(2)^2 - 12(2) + 7$

$= -5$

$y(4) = 3(4)^2 - 12(4) + 7$

$= 7$

Absolute min: $(2, -5)$

Absolute max: $(0, 7)$ and $(4, 7)$

b) $g(x) = 2x^3 - 3x^2 - 12x + 2, -3 \leq x \leq 3$

$g'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

critical #'s : $x_1 = 2 \quad x_2 = -1$

$g(-3) = 2(-3)^3 - 3(-3)^2 - 12(-3) + 2$
$= -43$

$g(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 2$
$= 9$

$g(2) = 2(2)^3 - 3(2)^2 - 12(2) + 2$
$= -18$

$g(3) = 2(3)^3 - 3(3)^2 - 12(3) + 2$
$= -7$

absolute min: $(-3, -43)$

absolute max: $(-1, 9)$

c) $f(x) = x^3 + x, 0 \leq x \leq 10$

$f'(x) = 3x^2 + 1$

$0 = 3x^2 + 1$

$x = \pm\sqrt{\frac{-1}{3}}$

So no critical #'s

$f(0) = 0$

$f(10) = 10^3 + 10$
$= 1010$

absolute min: $(0, 0)$

absolute max: $(10, 1010)$

3) Find and classify the critical points of each function as a local max, local min, or neither.

a) $y = 4x - x^2$

$y' = 4 - 2x$

$0 = 4 - 2x$

$x = 2$

$y(2) = 4(2) - (2)^2$

$y(2) = 4$

$(2, 4)$ is a critical point

$-\infty \quad 2 \quad \infty$

Test value	1	3
$f'(x)$	+	−
$f(x)$	inc. ↗	dec. ↘

local max

$\boxed{(2, 4) \text{ is a local MAX}}$

b) $f(x) = (x-1)^4$

$f'(x) = 4(x-1)^3(1)$

$0 = 4(x-1)^3$

$0 = (x-1)^3$

$0 = x-1$

$x = 1$

$f(1) = (1-1)^4$

$f(1) = 0$

(1,0) is a critical point

$-\infty$ 1 ∞

Test Value	0	2
$f'(x)$	−	+
$f(x)$	dec. ↘	inc. ↗

local min (1,0)

(1,0) is a local MIN

c) $g(x) = 2x^3 - 24x + 5$

$g'(x) = 6x^2 - 24$

$0 = 6(x^2-4)$

$0 = 6(x-2)(x+2)$

$x_1 = 2 \quad x_2 = -2$

$f(2) = -27 \quad f(-2) = 37$

critical points: (2,-27) and (-2,37)

$-\infty$ -2 2 ∞

Test	-3	0	3
$f'(x)$	+	−	+
$f(x)$	inc. ↗	dec. ↘	inc. ↗

local max local min

(-2,37) is a local MAX
(2,-27) is a local MIN

d) $y = \frac{1}{4}x^4 - \frac{2}{3}x^3$

$\frac{dy}{dx} = x^3 - 2x^2$

$0 = x^2(x-2)$

$x_1 = 0 \quad x_2 = 2$

$y(0) = 0 \quad y(2) = \frac{-4}{3}$

critical points: (0,0) and $(2, \frac{-4}{3})$

$-\infty$ 0 2 ∞

Test Value	-1	1	3
$f'(x)$	−	−	+
$f(x)$	dec ↘	dec. ↘	inc. ↗

local min

$(2, -\frac{4}{3})$ is a local MIN

(0,0) is neither

4)a) Find the critical numbers of $f(x) = 2x^3 - 3x^2 - 12x + 5$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

Critical #'s: $x_1 = 2 \quad x_2 = -1$

$f(2) = -15 \quad f(-1) = 12$

critical points: (2,-15) and (-1,12)

$f'(x) = 6(x-2)(x+1)$

b) Find any local extrema of $f(x)$.

$-\infty$ -1 2 ∞

Test	-2	0	3
$f'(x)$	+	−	+
$f(x)$	inc. ↗	dec. ↘	inc. ↗

local max local MIN

$(-1, 12)$ is a local MAX

$(2, -15)$ is a local MIN

c) Find the absolute extrema of $f(x)$ in the interval $[-2,4]$.

$f(-2) = 1$

$f(-1) = 12$

$f(2) = -15$

$F(4) = 37$

Absolute MIN: $(2, -15)$

Absolute MAX: $(4, 37)$

5) A section of rollercoaster is in the shape of $f(x) = -x^3 - 2x^2 + x + 15$, where x is between -2 and 2.

a) Find all local extrema and explain what portions of the rollercoaster they represent.

$f'(x) = -3x^2 - 4x + 1$

$0 = -3x^2 - 4x + 1$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(-3)(1)}}{2(-3)}$$

$$x = \frac{4 \pm 2\sqrt{7}}{-6} = \frac{2 \pm \sqrt{7}}{-3}$$

$x_1 \simeq -1.55$ $x_2 \simeq 0.22$

$f(-1.55) \simeq 12.37$ $F(0.22) \simeq 15.11$

critical points: $(-1.55, 12.37)$ and $(0.22, 15.11)$

$-\infty$ -1.55 0.22 ∞

Test	-2	0	1
$f'(x)$	−	+	−
$F(x)$	dec ↘	inc ↗	dec. ↘

The coaster starts going down a hill at $x = -2$, reaches a min at $(-1.55, 12.37)$, goes up to a max at $(0.22, 15.11)$, then continues down until $x = 2$.

b) Is the highest point of this section of the ride at the beginning, the end, or neither?

$F(-2) = 13$

$F(2) = 1$

∴ The absolute max is at $(0.22, 15.11)$; NOT at the beginning or end.

Answers:

1)a) $x = 3$ **b)** no critical numbers **c)** $x = -1,2$ **d)** $x = \frac{1}{4}$

2)a) absolute max at $(0,7)$ and $(4,7)$
absolute min at $(2,-5)$

b) absolute max at $(-1,9)$
absolute min at $(-3,-43)$

c) absolute max at $(10,1010)$
absolute min at $(0,0)$

3)a) $(2,4)$ is a local max **b)** $(1,0)$ is a local min **c)** $(-2,37)$ is a local max; $(2,-27)$ is a local min
d) $(0,0)$ is neither; $\left(2,-\frac{4}{3}\right)$ is a local min

4)a) $x = -1,2$ **b)** $(-1,12)$ is a local max; $(2,-15)$ is a local min **c)** $(2,-15)$ is the absolute min, $(4,37)$ is the absolute max

5)a) The coaster starts down a hill from $x = -2$, reaching a local min at the bottom of a hill at $(-1.55,12.37)$. It then increases height until it reaches a local max at the top of a hill at $(0.22,15.11)$. It then continues downward until $x = 2$.
b) The highest point is at $(0.22,15.11)$, not either of the endpoints.