

W2 - 6.4 - Power Law of Logarithms

MHF4U

Jensen

SOLUTIONS

1) Evaluate.

a) $\log_2 16^3$

$$= 3 \log_2 (2^4)$$

$$= 3(4)$$

$$= 12$$

b) $\log_4 8^2$

$$= \log_4 64$$

$$= \log_4 (4^3)$$

$$= 3$$

c) $\log 100^{-4}$

$$= -4 \log(10^2)$$

$$= -4(2)$$

$$= -8$$

d) $\log 0.1^{\frac{1}{2}}$

$$= \frac{1}{2} \log(10^{-1})$$

$$= \frac{1}{2}(-1)$$

$$= -\frac{1}{2}$$

e) $\log_2 \sqrt{8}$

$$= \frac{1}{2} \log_2 (2^3)$$

$$= \frac{1}{2}(3)$$

$$= \frac{3}{2}$$

f) $\log_3 (\sqrt[3]{81})^6$

$$= \log_3 (81^{\frac{1}{3}})^6$$

$$= \log_3 (81)^2$$

$$= 2 \log_3 (3^4)$$

$$= 2(4)$$

$$= 8$$

2) Solve for t to two decimal places.

a) $10 = 4^t$

$$\log_4 10 = t$$

$$\frac{\log 10}{\log 4} = t$$

$$t = 1.66$$

b) $5^t = 250$

$$\log_5 250 = t$$

$$\frac{\log 250}{\log 5} = t$$

$$t = 3.43$$

c) $2 = 1.08^t$

$$\log_{1.08} 2 = t$$

$$\frac{\log 2}{\log 1.08} = t$$

$$t = 9.01$$

d) $500 = 100(1.06)^t$

$$5 = 1.06^t$$

$$\log_{1.06} 5 = t$$

$$\frac{\log 5}{\log 1.06} = t$$

$$t = 27.62$$

3) An investment earns 7% interest, compounded annually. The amount, A , that the investment is worth as a function of time, t , in years, is given by $A(t) = 500(1.07)^t$.

a) Use the equation to determine the value of the investment after 4 years.

$$A(4) = 500(1.07)^4$$

$$= \$655.40$$

b) How long will it take for the investment to double in value?

$$1000 = 500(1.07)^t$$

$$2 = 1.07^t$$

$$\log_{1.07} 2 = t$$

$$\frac{\log 2}{\log 1.07} = t$$

$$t = 10.2 \text{ years}$$

4) Use the change of base formula to evaluate each of the following. Round to 3 decimal places.

a) $\log_3 23$

$$= \frac{\log 23}{\log 3}$$

$$= 2.854$$

b) $\log_6 20$

$$= \frac{\log 20}{\log 6}$$

$$= 1.672$$

c) $-\log_{12} 4$

$$= -\frac{\log 4}{\log 12}$$

$$= -0.558$$

d) $\log_{\frac{1}{2}} 30$

$$= \frac{\log 30}{\log(\frac{1}{2})}$$

$$= -4.907$$

5) Write each as a single logarithm

a) $\frac{\log 8}{\log 5}$

$$= \log_5 8$$

b) $\frac{\log 17}{\log 9}$

$$= \log_9 17$$

c) $\frac{\log(\frac{1}{2})}{\log(\frac{2}{3})}$

$$= \log_{\frac{2}{3}}(\frac{1}{2})$$

d) $\frac{\log(x+1)}{\log(x-1)}$

$$= \log_{(x-1)}(x+1)$$

6)a) Evaluate $\log_2 8^5$ without using the power law of logarithms.

$$= \log_2 (2^3)^5$$

$$= \log_2 (2^{15})$$

$$= 15$$

b) Evaluate the same expression by applying the power law of logarithms.

$$= 5 \cdot \log_2 (2^3)$$

$$= 5(3)$$

$$= 15$$

c) Which method do you prefer?

answers will vary

7) Solve for x , correct to 3 decimal places.

a) $2 = \log 3^x$

$$2 = x \cdot \log 3$$

$$\frac{2}{\log 3} = x$$

$$x = 4.192$$

b) $100 = 10 \log 1000^x$

$$10 = \log 1000^x$$

$$10 = x \log 1000$$

$$\frac{10}{\log 1000} = x$$

$$x = 3.333$$

c) $4 = \log_3 15^x$

$$3^4 = 15^x$$

$$81 = 15^x$$

$$\log 81 = \log 15^x$$

$$\log 81 = x \cdot \log 15$$

$$x = \frac{\log 81}{\log 15}$$

$$x = 1.623$$

ANSWER KEY

1) 12 b) 3 c) -8 d) $-\frac{1}{2}$ e) $\frac{3}{2}$ f) 8

2)a) 1.66 b) 3.43 c) 9.01 d) 27.62

3)a) \$655.40 b) 10.2 years

4)a) 2.854 b) 1.672 c) -0.558 d) -4.907

5)a) $\log_5 8$ b) $\log_9 17$ c) $\log_{\frac{2}{3}}(\frac{1}{2})$ d) $\log_{(x-1)}(x+1)$

6)a) 15 b) 15 c) answers will vary

7)a) 4.192 b) 3.333 c) 1.623