

### W3 - 4.5 Double Angle Formulas

MHF4U

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SOLUTIONS

1) Express each of the following as a single trig ratio.

a)  $2 \sin(5x) \cos(5x)$

$$= \sin[2(5x)]$$

$$= \sin(10x)$$

b)  $\cos^2 \theta - \sin^2 \theta$

$$= \cos(2\theta)$$

c)  $1 - 2 \sin^2(3x)$

$$= \cos[2(3x)]$$

$$= \cos(6x)$$

d)  $\frac{2 \tan(4x)}{1 - \tan^2(4x)}$

$$= \tan[2(4x)]$$

$$= \tan(8x)$$

e)  $4 \sin \theta \cos \theta$

$$= 2(2) \sin \theta \cos \theta$$

$$= 2 \sin(2\theta)$$

f)  $2 \cos^2 \frac{\theta}{2} - 1$

$$= \cos\left[2\left(\frac{\theta}{2}\right)\right]$$

$$= \cos \theta$$

2) Express each of the following as a single trig ratio and then evaluate

a)  $2 \sin 45^\circ \cos 45^\circ$

$$= \sin(2 \times 45^\circ)$$

$$= \sin 90^\circ$$

b)  $\cos^2 30^\circ - \sin^2 30^\circ$

$$= \cos(2 \times 30^\circ)$$

$$= \cos 60^\circ$$

c)  $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$

$$= \sin\left[2\left(\frac{\pi}{12}\right)\right]$$

$$= \sin\left(\frac{\pi}{6}\right)$$

d)  $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$

$$= \cos\left[2\left(\frac{\pi}{12}\right)\right]$$

$$= \cos\left(\frac{\pi}{6}\right)$$

e)  $1 - 2 \sin^2 \frac{3\pi}{8}$

$$= \cos\left[2\left(\frac{3\pi}{8}\right)\right]$$

$$= \cos\left(\frac{3\pi}{4}\right)$$

f)  $2 \tan 60^\circ \cos^2 60^\circ$

$$= 2 \left(\frac{\sin 60^\circ}{\cos 60^\circ}\right) \cos^2 60^\circ$$

$$= 2 \sin 60^\circ \cos 60^\circ$$

$$= \sin[2(60^\circ)]$$

$$= \sin 120^\circ$$

3) Use a double angle formula to rewrite each trig ratio

a)  $\sin(4\theta) = \sin[2(2\theta)]$

$$= 2 \sin(2\theta) \cos(2\theta)$$

b)  $\cos(3x) = \cos\left[2\left(\frac{3x}{2}\right)\right]$

$$= 2 \cos^2\left(\frac{3x}{2}\right) - 1$$

c)  $\tan x = \tan\left[2\left(\frac{x}{2}\right)\right]$

$$= \frac{2 \tan\left(\frac{x}{2}\right)}{1 - \tan^2\left(\frac{x}{2}\right)}$$

d)  $\cos(6\theta) = \cos[2(3\theta)]$

$$= \cos^2(3\theta) - \sin^2(3\theta)$$

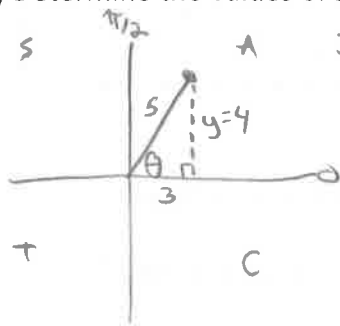
e)  $\sin x = \sin\left[2\left(\frac{x}{2}\right)\right]$

$$= 2 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)$$

f)  $\tan(5\theta) = \tan\left[2\left(\frac{5\theta}{2}\right)\right]$

$$= \frac{2 \tan\left(\frac{5\theta}{2}\right)}{1 - \tan^2\left(\frac{5\theta}{2}\right)}$$

- 4) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\cos \theta = \frac{3}{5}$  and  $0 \leq \theta \leq \frac{\pi}{2}$



$$3^2 + y^2 = 5^2$$

$$y^2 = 16$$

$$y = 4$$

$$\begin{aligned} \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{4}{5}\right) \left(\frac{3}{5}\right) \\ &= \frac{24}{25} \end{aligned}$$

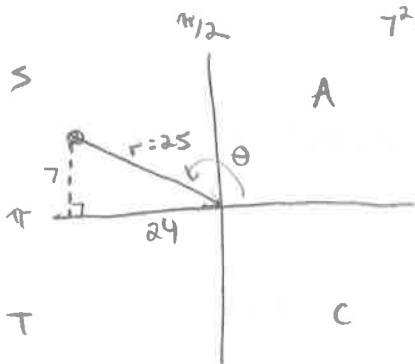
$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 \\ &= \frac{9}{25} - \frac{16}{25} \\ &= -\frac{7}{25} \end{aligned}$$

$$= \frac{\left(\frac{24}{25}\right)}{\left(-\frac{7}{25}\right)}$$

$$= -\frac{24}{7}$$

- 5) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\tan \theta = -\frac{7}{24}$  and  $\frac{\pi}{2} \leq \theta \leq \pi$



$$7^2 + 24^2 = r^2$$

$$625 = r^2$$

$$r = 25$$

$$\begin{aligned} \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{7}{25}\right) \left(-\frac{24}{25}\right) \\ &= -\frac{336}{625} \end{aligned}$$

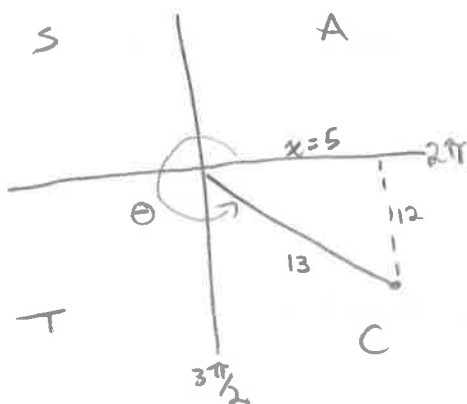
$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$= \frac{\left(-\frac{336}{625}\right)}{\left(\frac{527}{625}\right)}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= \left(-\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2 \\ &= \frac{576}{625} - \frac{49}{625} \\ &= \frac{527}{625} \end{aligned}$$

$$= -\frac{336}{527}$$

- 6) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\sin \theta = -\frac{12}{13}$  and  $\frac{3\pi}{2} \leq \theta \leq 2\pi$



$$x^2 + 12^2 = 13^2$$

$$x^2 = 25$$

$$x = 5$$

$$\begin{aligned} \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= 2 \left(-\frac{12}{13}\right) \left(\frac{5}{13}\right) \\ &= -\frac{120}{169} \end{aligned}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

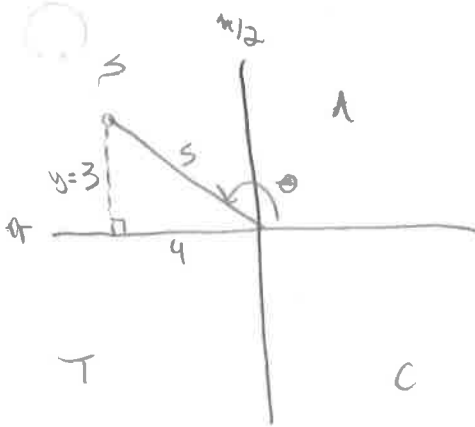
$$= \frac{\left(-\frac{120}{169}\right)}{\left(\frac{119}{169}\right)}$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2 \\ &= \frac{25}{169} - \frac{144}{169} \end{aligned}$$

$$= \frac{120}{119}$$

$$= -\frac{119}{169}$$

7) Determine the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ , given  $\cos \theta = -\frac{4}{5}$  and  $\frac{\pi}{2} \leq \theta \leq \pi$



$$x^2 + y^2 = s^2$$

$$y^2 = 9$$

$$y = 3$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{3}{5}\right) \left(-\frac{4}{5}\right)$$

$$= -\frac{24}{25}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$= \frac{\left(-\frac{24}{25}\right)}{\left(\frac{7}{25}\right)}$$

$$= -\frac{24}{7}$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

8) Determine the value of  $a$  in the equation  $2 \tan x - \tan(2x) + 2a = 1 - \tan(2x) \tan^2 x$

$$2 \tan x = \tan(2x) [1 - \tan^2 x] - 2a + 1$$

$$\frac{2 \tan x}{1 - \tan^2 x} = \frac{\tan(2x) [1 - \tan^2 x] - 2a + 1}{1 - \tan^2 x}$$

$$\tan(2x) = \tan(2x) + \frac{-2a + 1}{1 - \tan^2 x}$$

$$0 = -2a + 1$$

$$-1 = -2a$$

$$a = \frac{1}{2}$$

### Answer Key

1) a)  $\sin(10x)$  b)  $\cos(2\theta)$  c)  $\cos(6x)$  d)  $\tan(8x)$  e)  $2 \sin(2\theta)$  f)  $\cos \theta$

2) a)  $\sin 90^\circ; 1$  b)  $\cos 60^\circ; \frac{1}{2}$  c)  $\sin \frac{\pi}{6}; \frac{1}{2}$  d)  $\cos \frac{\pi}{6}; \frac{\sqrt{3}}{2}$  e)  $\cos \frac{3\pi}{4}; -\frac{1}{\sqrt{2}}$  f)  $\sin 120^\circ; \frac{\sqrt{3}}{2}$

3) a)  $2 \sin(2\theta) \cos(2\theta)$  b)  $2 \sin^2(1.5x) - 1$  c)  $\frac{2 \tan(0.5x)}{1 - \tan^2(0.5x)}$  d)  $\cos^2(3\theta) - \sin^2(3\theta)$  e)  $2 \sin(0.5x) \cos(0.5x)$  f)  $\frac{2 \tan(2.5\theta)}{1 - \tan^2(2.5\theta)}$

4)  $\sin(2\theta) = \frac{24}{25}$ ,  $\cos(2\theta) = -\frac{7}{25}$ ,  $\tan(2\theta) = -\frac{24}{7}$

5)  $\sin(2\theta) = -\frac{336}{625}$ ,  $\cos(2\theta) = \frac{527}{625}$ ,  $\tan(2\theta) = -\frac{336}{527}$

$\sin(2\theta) = -\frac{120}{169}$ ,  $\cos(2\theta) = -\frac{119}{169}$ ,  $\tan(2\theta) = \frac{120}{119}$

7)  $\sin(2\theta) = -\frac{24}{25}$ ,  $\cos(2\theta) = \frac{7}{25}$ ,  $\tan(2\theta) = -\frac{24}{7}$

8)  $a = \frac{1}{2}$