W3 – Concavity and the Second Derivative Unit 2

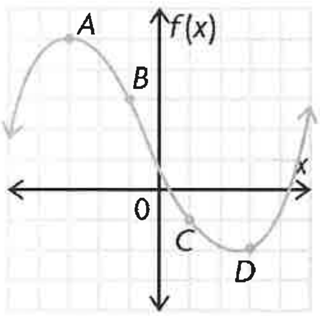
MCV4U

Jensen

SOLUTIONS

1) For each function, state whether the value of the of the second derivative is positive or negative at each of points A, B, C, and D.

a)



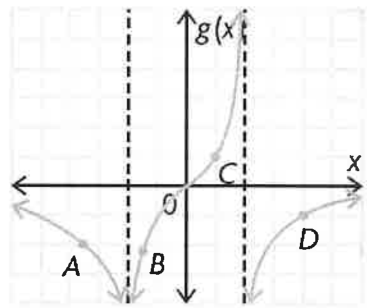
A: $f''(x) < 0$

B: $f''(x) < 0$

C: $f''(x) > 0$

D: $f''(x) > 0$

b)



A: $f''(x) < 0$

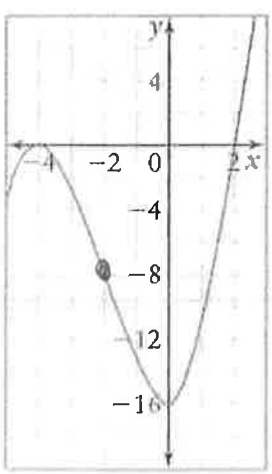
B: $f''(x) < 0$

C: $f''(x) > 0$

D: $f''(x) < 0$

2) For each graph, identify the intervals over which the graph is concave up and the intervals over which it is concave down.

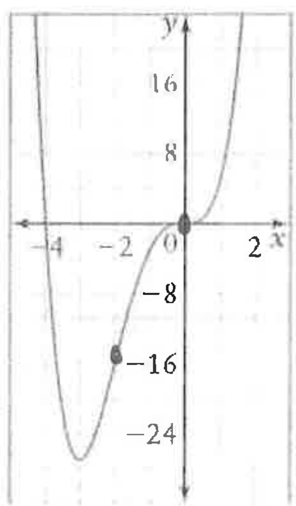
a)



concave up: $x > -2$

concave down: $x < -2$

b)

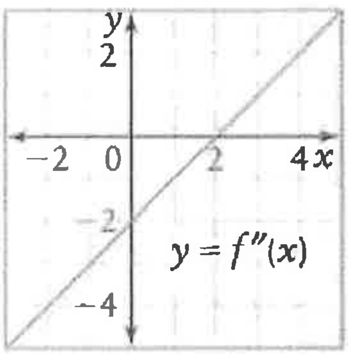


concave up: $x < -2$, $x > 0$

concave down: $-2 < x < 0$

3) Given each graph of $f''(x)$, state the intervals of concavity for the function $f(x)$. Also indicate where any points of inflection occur for $f(x)$.

a)

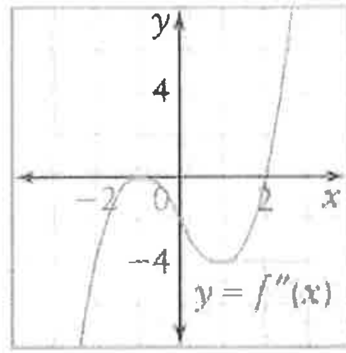


concave up: $x > 2$

concave down: $x < 2$

PoI when $x = 2$

b)



Concave up: $x > 2$

Concave down: $x < -1$, $-1 < x < 2$

PoI when $x = 2$

4) For each function, find the intervals of concavity and the coordinates of any points of inflection.

a) $y = 6x^2 - 7x + 5$

$y' = 12x - 7$

$y'' = 12$

$\therefore$ y is always concave up ; No PoI's.

b) $g(x) = -2x^3 + 12x^2 - 9$

$g'(x) = -6x^2 + 24x$

$g''(x) = -12x + 24$

$0 = -12x + 24$

$x = 2$

$g(2) = 23$

(2, 23) is a possible PoI

	$-\infty$ to 2	2 to ∞
Test	1	3
$f''(x)$	+	−
F(x)	concave up ∪	concave down ∩

concave up: $x < 2$

concave down: $x > 2$

POI: (2, 23)

5) For each function, find and classify all the critical points using the second derivative test.

a) $y = x^2 + 10x - 11$

$y' = 2x + 10$

$0 = 2x + 10$

$x = -5$

$y(-5) = -36$

$(-5, -36)$ is a critical point

2^{nd} Derivative Test:

$y'' = 2$

$y''(-5) = 2$

$\therefore$ y is concave up when $x = -5$

$(-5, -36)$ is a local min.

b) $f(x) = x^4 - 6x^2 + 10$

$f'(x) = 4x^3 - 12x$

$0 = 4x(x^2 - 3)$

$x_1 = 0 \quad x_2 = \sqrt{3} \quad x_3 = -\sqrt{3}$

$f(0) = 10 \quad f(\sqrt{3}) = 1 \quad F(-\sqrt{3}) = 1$

Critical points:

$(0, 10)$, $(\sqrt{3}, 1)$, and $(-\sqrt{3}, 1)$

2^{nd} derivative test: $f''(x) = 12x^2 - 12$

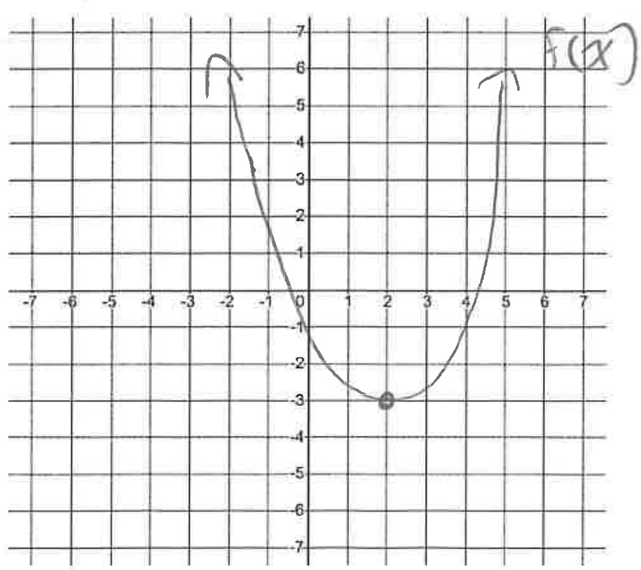
$f''(0) = -12$ concave down; $(0, 10)$ is a local max

$f''(\sqrt{3}) = 24$ concave up; $(\sqrt{3}, 1)$ is a local min

$f''(-\sqrt{3}) = 24$ concave up; $(-\sqrt{3}, 1)$ is a local min

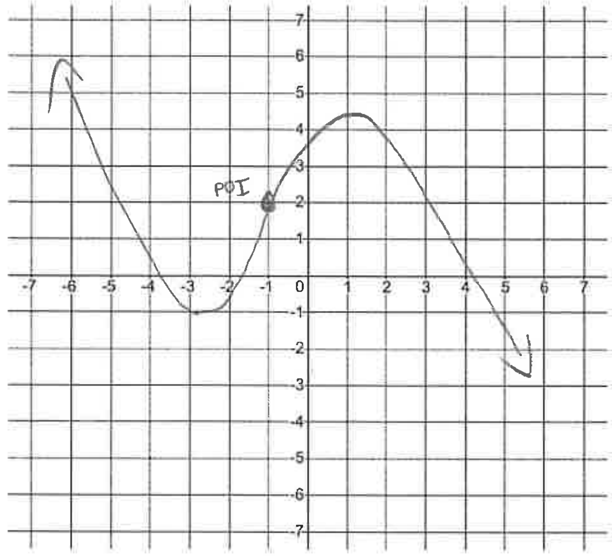
6) Sketch a graph of a function that satisfies each set of conditions

a) $f''(x) = 2$ for all x, $f'(2) = 0$, $f(2) = -3$



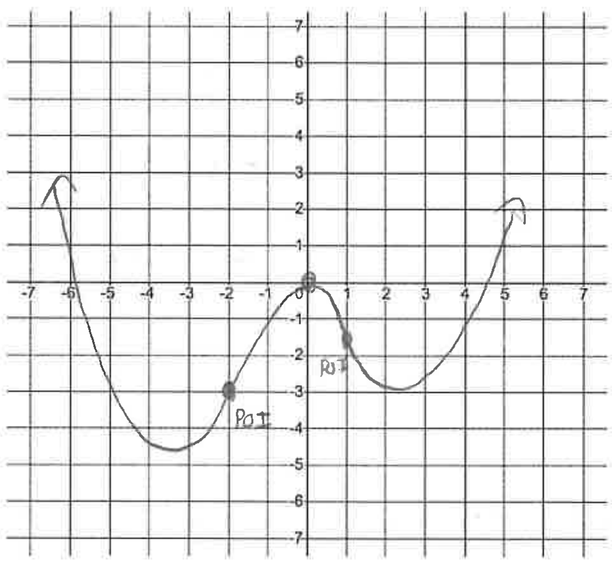
con. up   con. down

b) $f''(x) > 0$ when $x < -1$, $f''(x) < 0$ when $x > -1$, $f'(-1) = 1$, $f(-1) = 2$



con. down   con. up

c) $f''(x) < 0$ when $-2 < x < 1$, $f''(x) > 0$ when $x < -2$ and $x > 1$, $f(-2) = -3$, $f(0) = 0$



Answers:

1)a) A-neg, B-neg, C-pos, D-pos **b)** A-neg, B-neg, C-pos, D-neg

2)a) concave up: $x > -2$
concave down: $x < -2$

b) concave up: $x < -2$, $x > 0$
concave down: $-2 < x < 0$

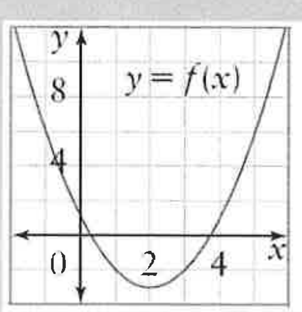
3)a) concave up: $x > 2$; concave down: $x < 2$; POI when $x = 2$

b) concave up: $x > 2$; concave down: $x < -1$ and $-1 < x < 2$; POI when $x = 2$

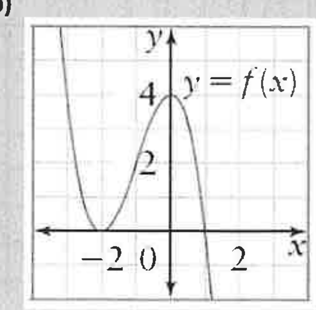
4)a) always concave up **b)** concave up: $x < 2$; concave down: $x > 2$; POI at (2,23).

5)a) $(-5, -36)$ is a local min point **b)** $(-\sqrt{3}, 1)$ and $(\sqrt{3}, 1)$ are local mins, (0,10) is a local max

6)a)



b)



c)