

**W3 – Multiplication of a Vector by a Scalar**

MCV4U

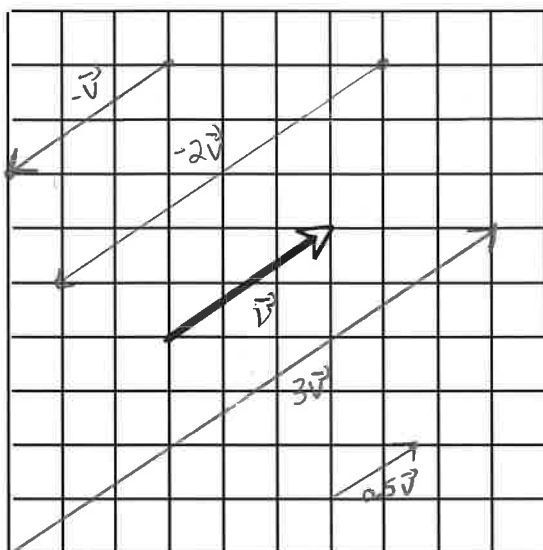
Jensen

Unit 4

SOLUTIONS

1) Draw representatives of the following vectors on the grid provided.

- a)  $3\vec{v}$
- b)  $0.5\vec{v}$
- c)  $-2\vec{v}$
- d)  $-\vec{v}$



2) Simplify each of the following algebraically.

a)  $\vec{a} + 2\vec{a} + 4\vec{a}$

$= 7\vec{a}$

b)  $7\vec{u} + 5\vec{v} - 2\vec{u} + 8\vec{v}$

$= 5\vec{u} + 13\vec{v}$

c)  $2(\vec{u} + \vec{v}) - 3(\vec{u} - 2\vec{v})$

$= 2\vec{u} + 2\vec{v} - 3\vec{u} + 6\vec{v}$   
 $= -\vec{u} + 8\vec{v}$

d)  $7\vec{u} + 5\vec{v} - 2(\vec{u} - \vec{v}) + 2\vec{u}$

$= 7\vec{u} + 5\vec{v} - 2\vec{u} + 2\vec{v} + 2\vec{u}$   
 $= 7\vec{u} + 7\vec{v}$

e)  $-3(\vec{u} + \vec{v}) + 2(\vec{u} - \vec{v})$

$= -3\vec{u} - 3\vec{v} + 2\vec{u} - 2\vec{v}$   
 $= -\vec{u} - 5\vec{v}$

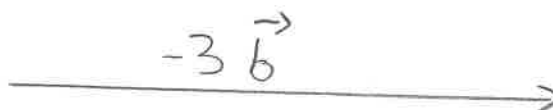
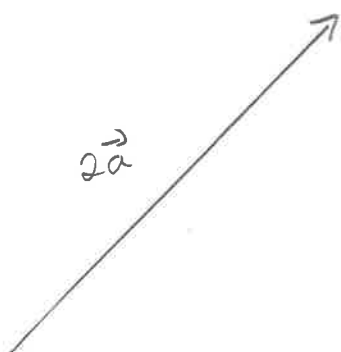
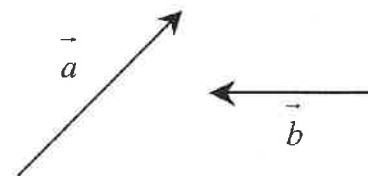
f)  $6(\vec{u} + 2\vec{v}) - 5(\vec{u} - 3\vec{v})$

$= 6\vec{u} + 12\vec{v} - 5\vec{u} + 15\vec{v}$   
 $= \vec{u} + 27\vec{v}$

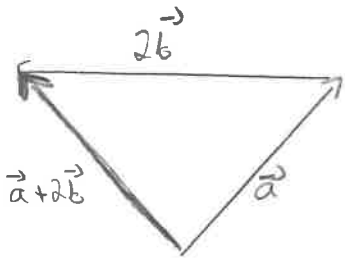
3) For the vectors  $\vec{a}$  and  $\vec{b}$  shown, draw and label...

a)  $2\vec{a}$

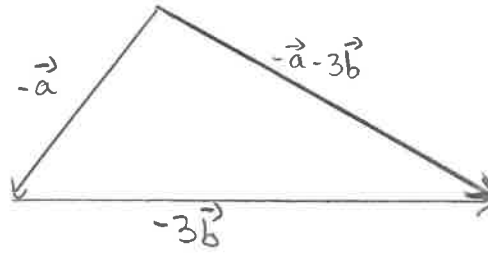
b)  $-3\vec{b}$



c)  $\vec{a} + 2\vec{b}$

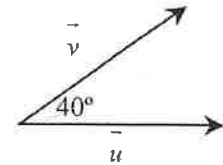
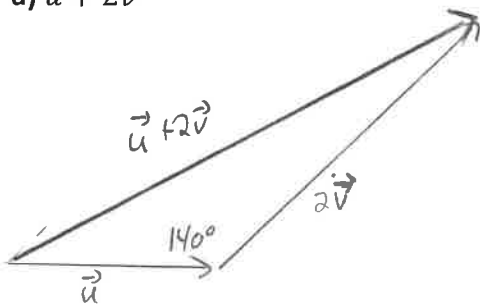


d)  $-\vec{a} - 3\vec{b}$

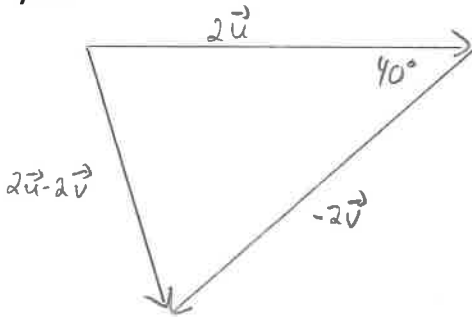


4) Two vectors  $\vec{u}$  and  $\vec{v}$  make an angle of  $40^\circ$  with each other. Construct each vector sum or difference.

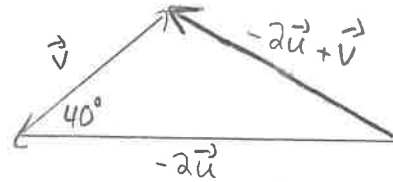
a)  $\vec{u} + 2\vec{v}$



b)  $2\vec{u} - 2\vec{v}$



c)  $-2\vec{u} + \vec{v}$



5) In parallelogram ABCD, opposite sides are parallel and equal,  $\overline{BP} = \overline{PA}$ , and  $\overline{AQ} = \overline{QD}$ . Let  $\overline{BP} = \vec{u}$  and  $\overline{AQ} = \vec{v}$ . Express the following vectors in terms of  $\vec{u}$  and  $\vec{v}$ .

a)  $\overline{AD}$

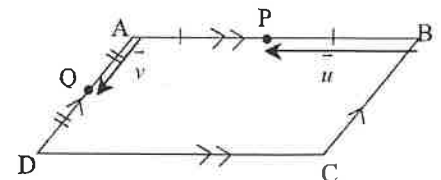
$= 2\vec{v}$

b)  $\overline{PA}$

$= \vec{u}$

c)  $\overline{CD}$

$= 2\vec{u}$



d)  $\overline{PQ}$

$= \vec{u} + \vec{v}$

e)  $\overline{BD}$

$= 2\vec{v} + 2\vec{u}$

f)  $\overline{PD}$

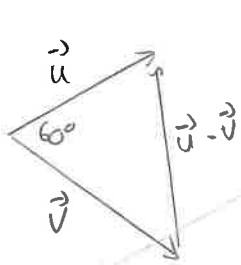
$= \vec{u} + 2\vec{v}$

g)  $\overline{AC}$

$= 2\vec{v} - 2\vec{u}$

6) Given that  $|\vec{u}| = 8$  and  $|\vec{v}| = 10$  and the angle between  $\vec{u}$  and  $\vec{v}$  is  $60^\circ$  determine:

a)  $|\vec{u} - \vec{v}|$

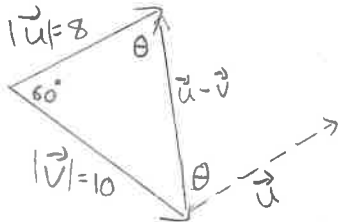


$$|\vec{u} - \vec{v}|^2 = 8^2 + 10^2 - 2(8)(10)\cos(60^\circ)$$

$$|\vec{u} - \vec{v}| = \sqrt{84}$$

$$|\vec{u} - \vec{v}| = 2\sqrt{21}$$

b) the direction of  $\vec{u} - \vec{v}$  relative to  $\vec{u}$

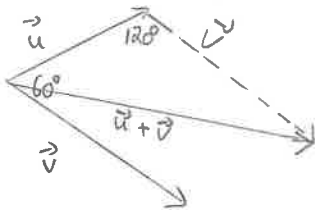


$$\frac{\sin \theta}{10} = \frac{\sin 60}{2\sqrt{21}}$$

$$\theta \approx 70.9^\circ$$

$70.9^\circ$  between  $\vec{u}$  and  $\vec{u} - \vec{v}$

c) the unit vector in the direction of  $\vec{u} + \vec{v}$



$$|\vec{u} + \vec{v}|^2 = 8^2 + 10^2 - 2(8)(10)\cos(120)$$

$$|\vec{u} + \vec{v}| = \sqrt{244}$$

$$|\vec{u} + \vec{v}| = 2\sqrt{61}$$

$$\text{unit vector} = \frac{1}{2\sqrt{61}} (\vec{u} + \vec{v})$$

d)  $|5\vec{u} + 2\vec{v}|$

$$|5\vec{u}| = 40$$

$$|2\vec{v}| = 20$$

$$|5\vec{u} + 2\vec{v}|^2 = 40^2 + 20^2 - 2(40)(20)\cos(120)$$

$$|5\vec{u} + 2\vec{v}| = \sqrt{2800}$$

$$|5\vec{u} + 2\vec{v}| = 20\sqrt{7}$$

7)  $|\vec{v}| = 2$ . Draw the following factors and express each of them as a scalar multiple of  $\vec{v}$ .

a) A vector in the same direction as  $\vec{v}$  with twice its magnitude

b) a vector in the same direction as  $\vec{v}$  with one half its magnitude

c) a vector in the opposite direction as  $\vec{v}$  with two-thirds its magnitude

d) a vector in the opposite direction as  $\vec{v}$  with twice its magnitude

e) a unit vector in the same direction as  $\vec{v}$

a)

$$2\vec{v}$$

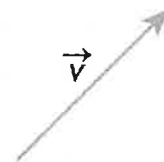
$$0.5\vec{v}$$

$$-\frac{2}{3}\vec{v}$$

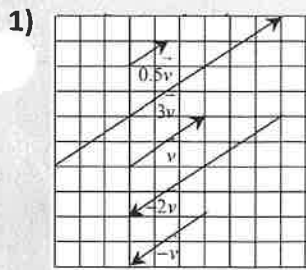
$$-2\vec{v}$$

e)

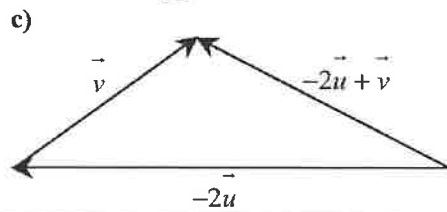
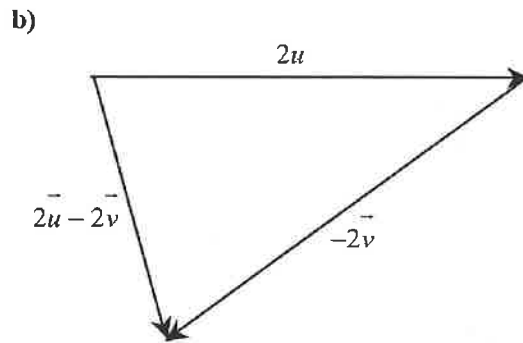
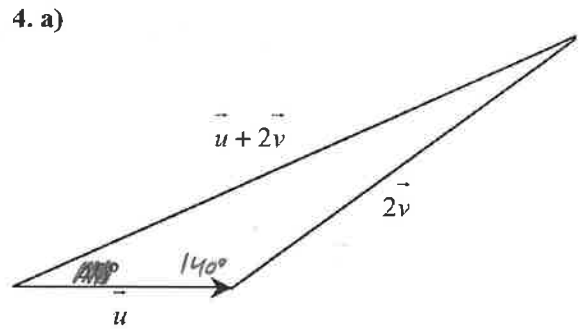
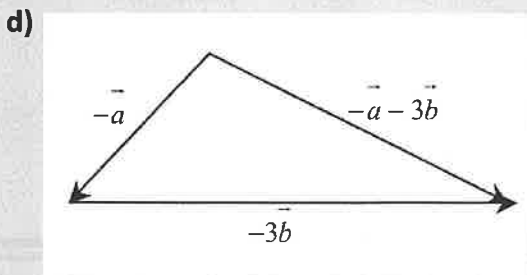
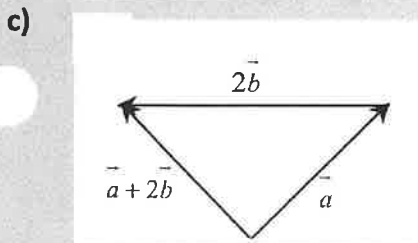
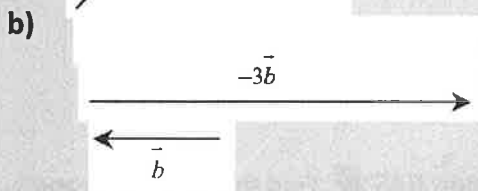
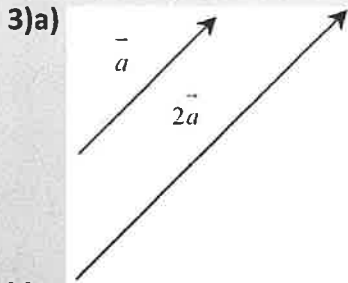
$$\frac{1}{|\vec{v}|}\vec{v}$$



Answers:



2)a)  $7\vec{a}$  b)  $5\vec{u} + 13\vec{v}$  c)  $-\vec{u} + 8\vec{v}$  d)  $7\vec{u} + 7\vec{v}$  e)  $-\vec{u} - 5\vec{v}$  f)  $\vec{u} + 27\vec{v}$



5)a)  $2\vec{v}$  b)  $\vec{u}$  c)  $2\vec{u}$  d)  $\vec{u} + \vec{v}$  e)  $2\vec{u} + 2\vec{v}$  f)  $\vec{u} + 2\vec{v}$  g)  $2\vec{v} - 2\vec{u}$

6)a)  $2\sqrt{21}$  b)  $71^\circ$  c)  $\frac{1}{2\sqrt{6}}(\vec{u} + \vec{v})$  d)  $20\sqrt{7}$

