W4 – Equations of Planes in Scalar Form Unit 6

MCV4U

Jensen

1) Write the scalar equation of each plane given the normal $\vec{n}$ and a point P on the plane.

a) $\vec{n}$ = [1, –3, –4], P(–2, 5, 7)

b) $\vec{n}$ = [0, –3, 5], P(8, –7, 3)

2) Write a scalar equation of each plane, given its vector or parametric equation.

a) $[x\ y\ z] = [7, -3, 4] + s[1, -2, -3] + t[4, -3, 1]$

b) $\pi_1 \begin{cases} x = 4 + t + 2s \\ y = 3 - 2t - 3s \\ z = -1 + 3t + s \end{cases}$

3) For each situation, write the vector and scalar equations of the plane.

a) has normal $\vec{n} = (7, 9, -1)$ and includes the point $(3, -2, 4)$

b) contains direction vectors $\vec{a}$= [–1, 2, 8] and $\vec{b}$= [2, –1, 3] and includes the point (2, –7, 8)

c) parallel to the xz-plane and includes the point (7, 8, –1)

d) contains the points (3, 8, –1), (–8, 9, –4), and (1, –3, 2)

e) contains the line [*x, y, z*] = [4, –3, –2] + *s*[3, –2, 1] and parallel to the line defined by the parametric equations $\ell \begin{cases} x = 5 + 3s \\ y = 1 - s \\ z = 2 + 4s \end{cases}$

f) contains the point (2, –1, 8) and perpendicular to the line $[x, y, z] = [1, -2, -3] + s[5, -4, 7]$

g) parallel to the plane $-3x + 2y + 5z + 8 = 0$ and includes the point $(5, -7, 8)$

h) contains the lines $[x, y, z] = [4, -1, 0] + s[-2, 1, 3]$ and $[x, y, z] = [-2, 4, 3] + s[-6, 5, 7]$

4) Determine the angle between the planes $x + 2y - 3z - 4 = 0$ and $x + 2y - 1 = 0$. *(Hint: the angle between 2 intersecting planes should be equal to the angle between their normals)*

5) Write the vector equation and parametric equations of the plane: $3x - y + 2z - 4 = 0$

6) Find a vector that is normal and one that is parallel to each plane.

a) $x + 2y + 2z - 5 = 0$ **b)** $5x + 2z = 7$ **c)** $5y = 8$

7) Consider the plane $-x + 4y + 2z + 6 = 0$

a) Determine a normal vector, $\vec{n}$, to the plane.

b) Determine the coordinates of two points, S and T, on the plane.

c) Determine $\overrightarrow{ST}$

d) Show that $\overrightarrow{ST}$ is perpendicular to $\vec{n}$.

8) State the scalar equation of each of the following

a) the xy-plane **b)** the xz-plane **c)** the yz-plane

ANSWER KEY:

1. a) $x - 3y - 4z + 45 = 0$ **b)** $-3y + 5z - 36 = 0$ **2. a)** $-11x - 13y + 5z + 18 = 0$ **b)** $7x + 5y + z - 42 = 0$
3. a) $7x + 9y - z + 1 = 0$ **b)** $14x + 19y - 3z + 129 = 0$ **c)** $y - 8 = 0$ **d)** $-30x + 39y + 123z - 99 = 0$
e) $-7x - 9y + 3z + 7 = 0$ **f)** $5x - 4y + 7z - 70 = 0$ **g)** $-3x + 2y + 5z - 11 = 0$ **h)** $2x + y + z - 7 = 0$
4. $53.3°$
5. $[x, y, z] = [0, 0, 2] + t[1, 3, 0] + s[0, 2, 1]$, parametric: $x = t, y = 3t + 2s, z = 2 + s$
6.a) perpendicular: $[2,4,4]$; parallel: [-2,0,1] **b)** perpendicular: [5,0,2], parallel: [-2,0,5] **c)** perpendicular: [0,5,0]; parallel: [2,0,1]
7.a) $\vec{n} = [-1,4,2]$ **b)** $S(0,0,-3), T(6,0,0)$ **c)** $\overrightarrow{ST} = [6,0,3]$ **d)** $\overrightarrow{ST} \cdot \vec{n} = 0$
8.a) $z = 0$ **b)** $y = 0$ **c)** $x = 0$