**W4 – 7.1/7.2 – Solving Exponential Equations**
**MHF4U**
***Jensen***

SOLUTIONS

**1)** Write each expression with base 2.

**a)** $4^6$

$= (2^2)^6$

$= 2^{12}$

**b)** $8^3$

$= (2^3)^3$

$= 2^9$

**c)** $\left(\frac{1}{8}\right)^2$

$= (2^{-3})^2$

$= 2^{-6}$

**d)** 14

$2^x = 14$

$x = \log_2 14$

$x = \frac{\log 14}{\log 2}$

$\therefore 2^{\frac{\log 14}{\log 2}} = 14$

**2)** Write each expression as a power of 4.

**a)** $\left(\sqrt{16}\right)^3$

$= 4^3$

**b)** $\sqrt[3]{16}$

$= 16^{1/3}$

$= (4^2)^{1/3}$

$= 4^{2/3}$

$4^x = 128$

$x = \log_4 128$

$x = \frac{\log 128}{\log 4}$

$x = 3.5$

$x = \frac{7}{2}$

**c)** $\sqrt{64} \times \left(\sqrt[4]{128}\right)^3$

$= 64^{1/2} \times 128^{3/4}$

$= (4^3)^{1/2} \times (4^{7/2})^{3/4}$

$= 4^{3/2} \times 4^{21/8}$

$= 4^{33/8}$

**3)** Solve each equation

**a)** $2^{4x} = 4^{x+3}$

$2^{4x} = (2^2)^{x+3}$

$2^{4x} = 2^{2x+6}$

$4x = 2x + 6$

$2x = 6$

$\boxed{x = 3}$

**b)** $3^{w+1} = 9^{w-1}$

$3^{w+1} = (3^2)^{w-1}$

$3^{w+1} = 3^{2w-2}$

$w + 1 = 2w - 2$

$\boxed{3 = w}$

**c)** $4^{3x} = 8^{x-3}$

$(2^2)^{3x} = (2^3)^{x-3}$

$2^{6x} = 2^{3x-9}$

$6x = 3x - 9$

$3x = -9$

$\boxed{x = -3}$

**d)** $125^{2y-1} = 25^{y+4}$

$(5^3)^{2y-1} = (5^2)^{y+4}$

$6y - 3 = 2y + 8$

$4y = 11$

$\boxed{y = \frac{11}{4}}$

**4)** Consider the equation $10^{2x} = 100^{2x-5}$

**a)** Solve this equation by expressing both sides as powers of a common base.

$10^{2x} = (10^2)^{2x-5}$

$10^{2x} = 10^{4x-10}$

$2x = 4x - 10$

$10 = 2x$

$\boxed{5 = x}$

**b)** Solve the same equation by taking the common logarithm of both sides.

$\log 10^{2x} = \log 100^{2x-5}$

$2x \log 10 = (2x - 5) \log 100$

$2x(1) = (2x - 5)(2)$

$2x = 4x - 10$

$10 = 2x$

$\boxed{x = 5}$

**5)** Solve $2^{3x} > 4^{x+1}$

$2^{3x} > (2^2)^{x+1}$

$2^{3x} > 2^{2x+2}$

$3x > 2x+2$

$\boxed{x > 2}$

**6)** Solve for $t$. Round answers to 2 decimal places.

**a)** $2 = 1.07^t$

$\log_{1.07} 2 = t$

$\frac{\log 2}{\log 1.07} = t$

$\boxed{t = 10.24}$

**b)** $\frac{100}{10} = \frac{10(1.04)^t}{10}$

$10 = 1.04^t$

$\log 10 = \log 1.04^t$

$\log 10 = t \cdot \log 1.04$

$t = \frac{\log 10}{\log 1.04}$

$\boxed{t = 58.71}$

**c)** $15 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

$\log_{1/2}(15) = \frac{t}{4}$

$\frac{\log(15)}{\log(\frac{1}{2})} = \frac{t}{4}$

$4\left[\frac{\log(15)}{\log(0.5)}\right] = t$

$\boxed{t = -15.63}$

**7)** Solve each equation. Round answers to 3 decimal places.

**a)** $2^x = 3^{x-1}$

$\log 2^x = \log 3^{x-1}$

$x \log 2 = (x-1)\log 3$

$x \log 2 = x \log 3 - \log 3$

$\log 3 = x \log 3 - x \log 2$

$\log 3 = x(\log 3 - \log 2)$

$x = \frac{\log 3}{\log 3 - \log 2}$

$\boxed{x = 2.71}$

**b)** $5^{x-2} = 4^x$

$\log 5^{x-2} = \log 4^x$

$(x-2)\log 5 = x \log 4$

$x \log 5 - 2\log 5 = x \log 4$

$x \log 5 - x \log 4 = 2 \log 5$

$x(\log 5 - \log 4) = 2 \log 5$

$x = \frac{2\log 5}{\log 5 - \log 4}$

$\boxed{x = 14.425}$

**c)** $7^{2x+1} = 4^{x-2}$

$\log 7^{2x+1} = \log 4^{x-2}$

$(2x+1)\log 7 = (x-2)\log 4$

$2x \log 7 + \log 7 = x \log 4 - 2\log 4$

$2x \log 7 - x \log 4 = -2\log 4 - \log 7$

$x(2\log 7 - \log 4) = -2\log 4 - \log 7$

$x = \frac{-2\log 4 - \log 7}{2\log 7 - \log 4}$

$\boxed{x = -1.883}$

**8)** Solve $2^{2x} + 2^x - 6 = 0$ using the quadratic formula. Clearly identify any extraneous roots.

$(2^x)^2 + (2^x) - 6 = 0$

Let $k = 2^x$

$k^2 + k - 6 = 0$

$(k+3)(k-2) = 0$

$k = -3, 2$

CASE 1

$-3 = 2^x$

$\log_2(-3) = x$

∴ NO SOLUTIONS

CASE 2

$2 = 2^x$

$\boxed{x = 1}$

**9)** Solve $8^{2x} - 2(8^x) - 5 = 0$ using the quadratic formula. Clearly identify any extraneous roots.

$(8^x)^2 - 2(8^x) - 5 = 0$

Let $k = 8^x$

$k^2 - 2k - 5 = 0$

$k = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$

$k = \frac{2 \pm \sqrt{24}}{2}$

$k = \frac{2 \pm 2\sqrt{6}}{2}$

$k = \frac{2(1 \pm \sqrt{6})}{2}$

Case 1

$8^x = 1 + \sqrt{6}$

$\log_8(1+\sqrt{6}) = x$

$\frac{\log(1+\sqrt{6})}{\log 8} = x$

$x = 0.595$

Case 2

$8^x = 1 - \sqrt{6}$

↑ NO SOLUTIONS

**10)** Use the decay equation for polonium-218, $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{3.1}}$, $A$ is the amount remaining after $t$ minutes and $A_0$ is the initial amount.

**a)** How much will remain after 90 seconds from an initial sample of 50 mg?

$A(1.5) = 50\left(\frac{1}{2}\right)^{1.5/3.1}$

$A(1.5) = 35.75$ mg.

**b)** How long will it take for this sample to decay to 10% of its initial amount of 50 mg?

$5 = 50\left(\frac{1}{2}\right)^{t/3.1}$

$0.1 = \left(\frac{1}{2}\right)^{t/3.1}$

$\log_{1/2}(0.1) = \frac{t}{3.1}$

$\frac{\log 0.1}{\log\left(\frac{1}{2}\right)} = \frac{t}{3.1}$

$3.321928095 = \frac{t}{3.1}$

$t = 10.298$ minutes

**11)** A 20-mg sample of thorium-233 decays to 17 mg after 5 minutes.

**a)** What is the half-life of thorium-233?

$17 = 20\left(\frac{1}{2}\right)^{5/h}$

$0.85 = \left(\frac{1}{2}\right)^{5/h}$

$\log_{1/2}(0.85) = \frac{5}{h}$

$\frac{\log(0.85)}{\log(0.5)} = \frac{5}{h}$

$h = \frac{5\log(0.5)}{\log(0.85)}$

$h = 21.325$ minutes

**b)** How long will it take this sample to decay to 1 mg?

$1 = 20\left(\frac{1}{2}\right)^{t/21.3}$

$0.05 = \left(\frac{1}{2}\right)^{t/21.3}$

$\log_{1/2}(0.05) = \frac{t}{21.3}$

$\frac{\log(0.05)}{\log(0.5)} = \frac{t}{21.3}$

$4.321928095 = \frac{t}{21.3}$

$t = 92.06$ min

**ANSWER KEY**

**1)a)** $2^{12}$ **b)** $2^{9}$ **c)** $2^{-6}$ **d)** $2^{\frac{\log 14}{\log 2}}$ **2)a)** $4^{3}$ **b)** $4^{\frac{2}{3}}$ **c)** $4^{\frac{33}{8}}$ **3)a)** 3 **b)** 3 **c)** -3 **d)** $\frac{11}{4}$

**4)a)** 5 **b)** 5 **5)** $x > 2$ **6)a)** 10.24 **b)** 58.71 **c)** -15.63 **7)a)** 2.710 **b)** 14.425 **c)** -1.883

**8)** $x = 1$ is the only solution; $2^x = -3$ or $x = \frac{\log(-3)}{\log 2}$ is an extraneous root

, $x = \frac{\log(1+\sqrt{6})}{\log 8} \cong 0.6$ is the only solution; $8^x = 1 - \sqrt{6}$ or $x = \frac{\log(1-\sqrt{6})}{\log 8}$ is an extraneous root

**10)a)** 35.75 mg **b)** 10.3 min **11)a)** 21.3 min **b)** 92.06 min